2017 ICNT Program at FRIB, FRIB-MSU, East Lansing, Michigan, March 22 – April 12, 2017

RECENT PROGRESS IN HIGH-PRECISION CHIRAL NUCLEAR FORCES

R. Machleidt

University of Idaho

OUTLINE

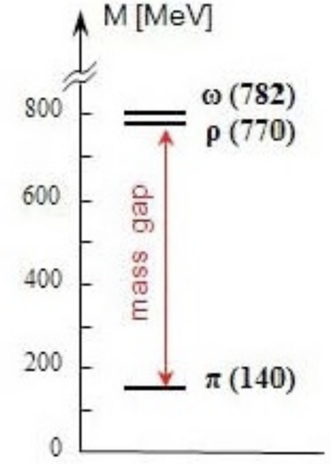
- Current status & current issues
- How to address the open issues?
- Consistent interactions up to N4LO
- Keeping the error budget low
- Conclusions

3

CURRENT STATUS

Motivation for the chiral EFT approach

- QCD at low energy is strong.
- Quarks and gluons are confined into colorless hadrons.
- Nuclear forces are residual forces (similar to van der Waals forces)
- Separation of scales



- **Calls for an EFT:**

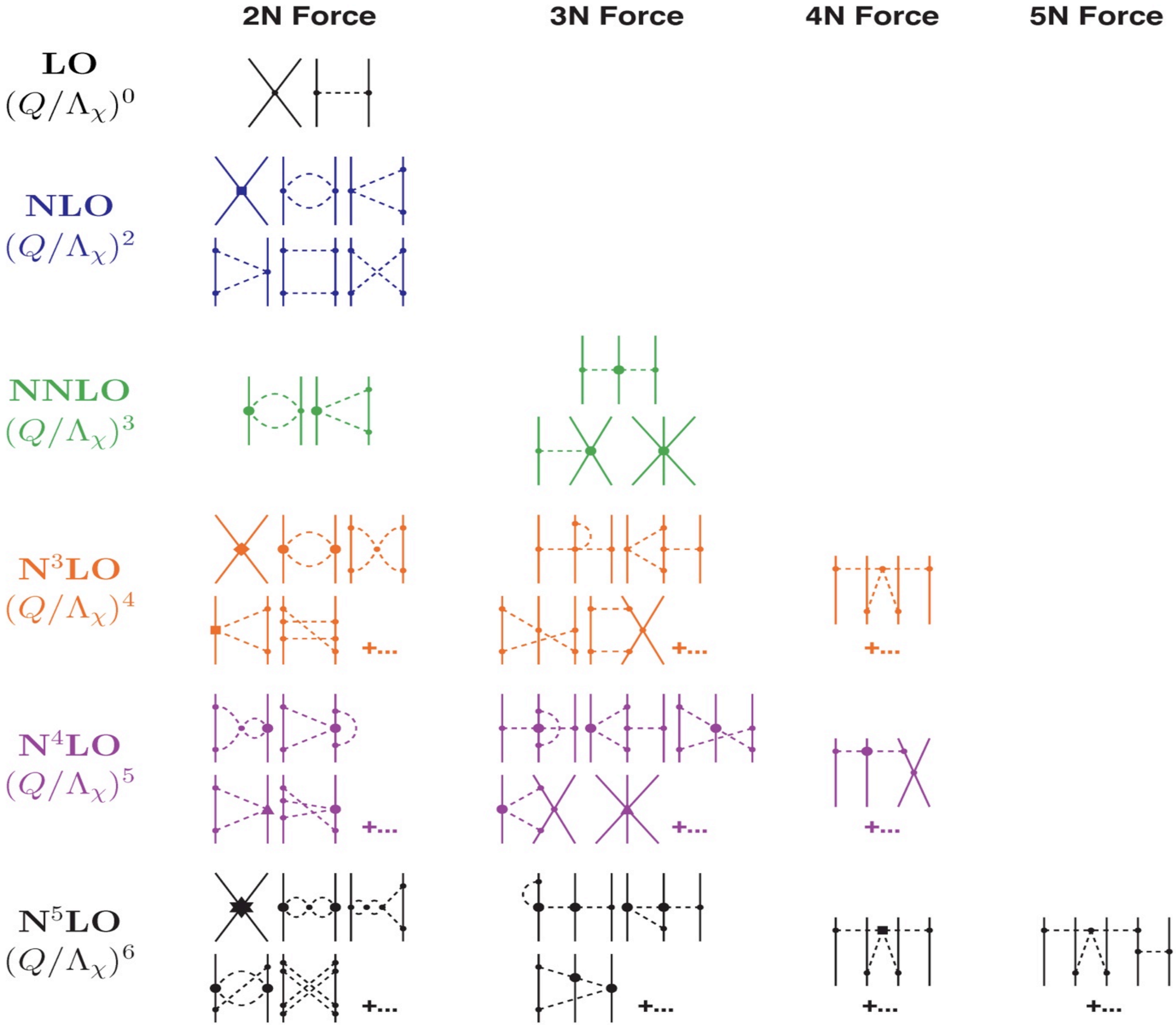
  soft scale: $Q \approx m_\pi$, hard scale: $\Lambda_\chi \approx m_\rho$; pions and nucleons are relevant d.o.f.
- **Low-momentum expansion: $(Q/\Lambda_\chi)^\nu$**

  with ν bounded from below.
- **Most general Lagrangian consistent with all symmetries of low-energy QCD, particularly, chiral symmetry which is spontaneously broken.**
- **Weakly interacting Goldstone bosons = pions.**
- **π-π and π-N perturbatively**
- **NN has bound states:**

  (i) NN potential perturbatively

  (ii) apply nonpert. in LS equation.

  (Weinberg)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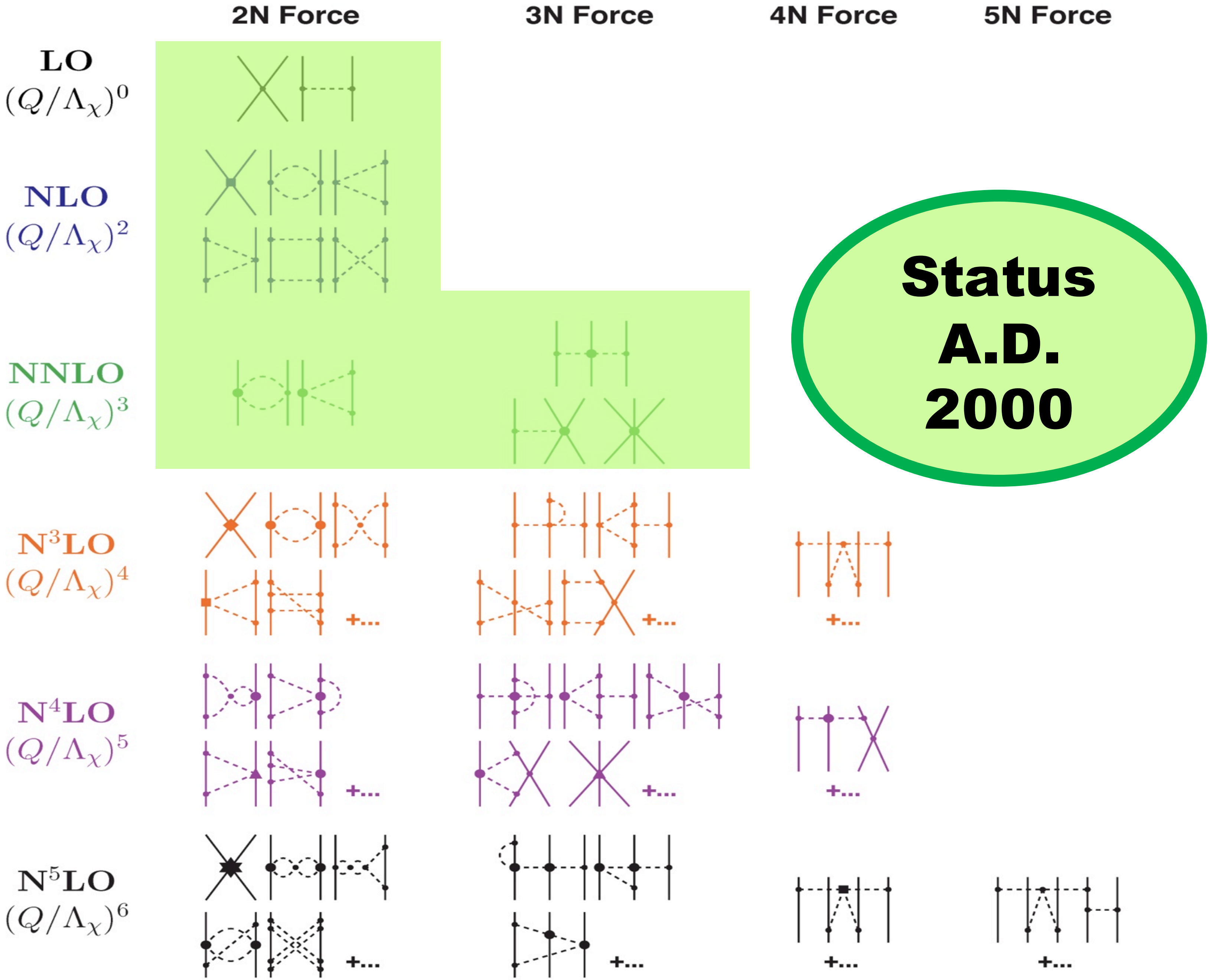
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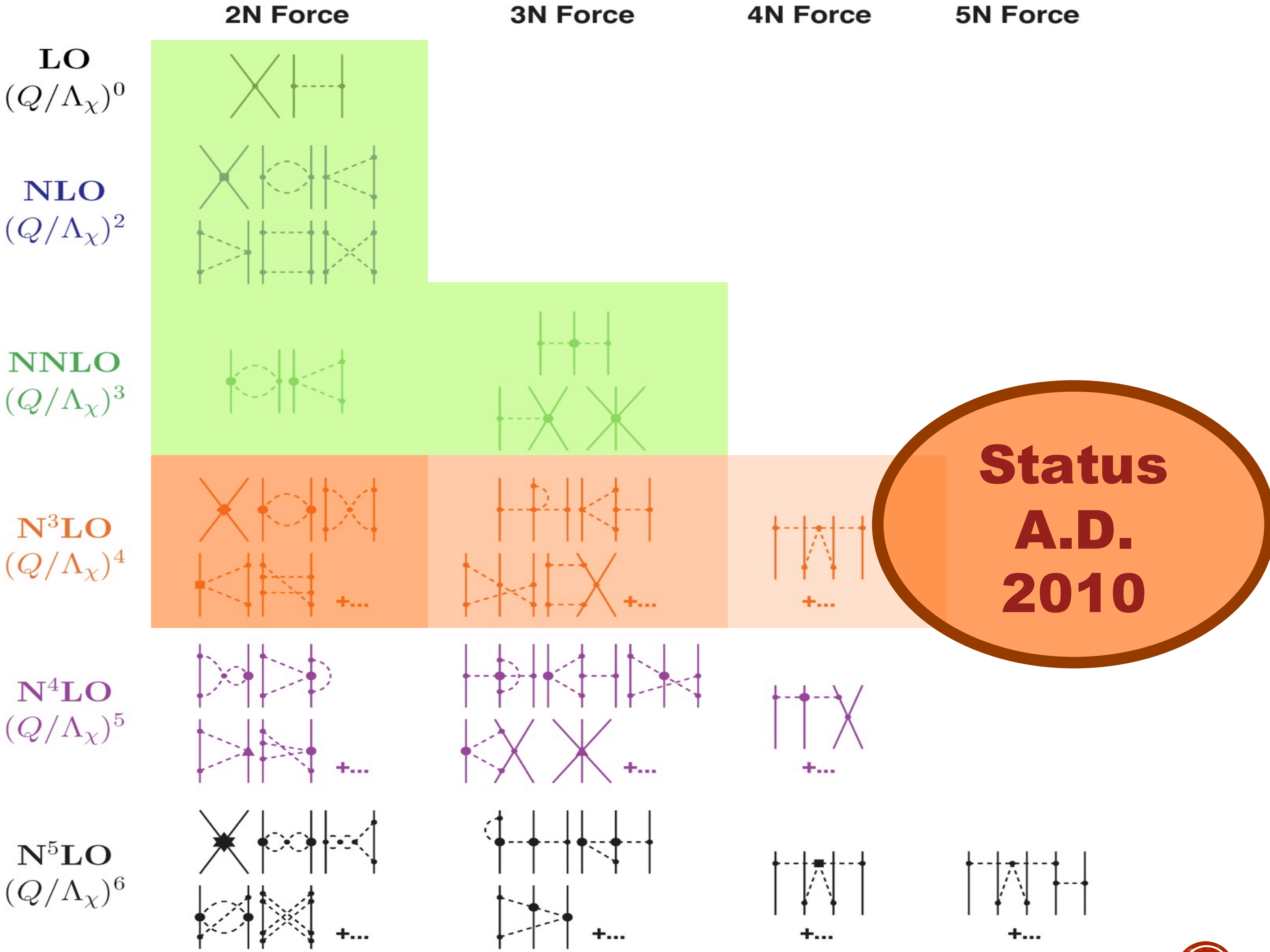
+...
Status
A.D.
2000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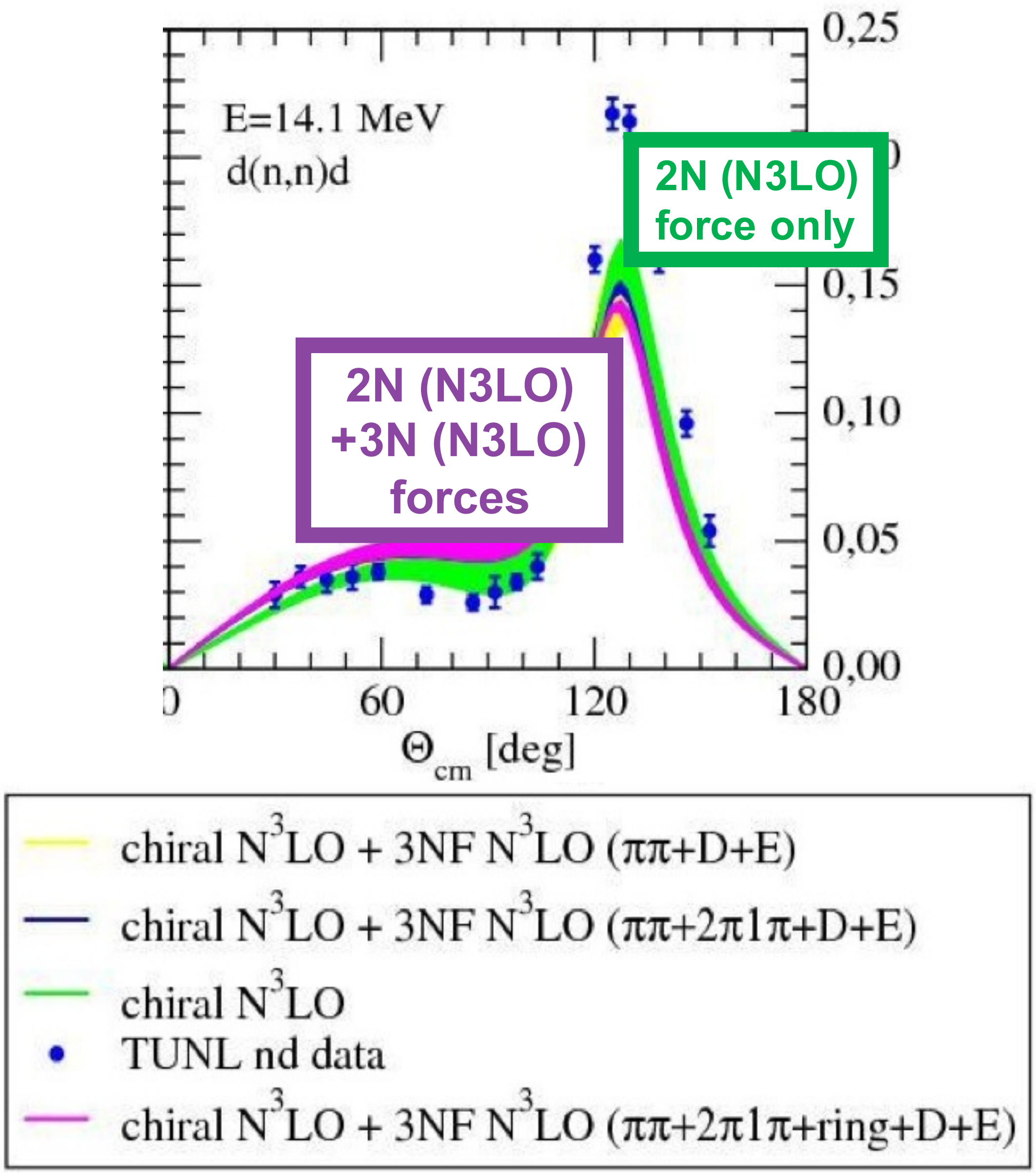
WHAT HAVE WE ACHIEVED WITH THOSE FORCES?

- **There has been some success (ground state of 10B, drip lines, nuclear matter saturation, orbit evolution, etc.), but some persistent problems remain.**
- **In the few-body sector: Ay puzzle, N-d break-up, …**

N-d A_y calculations by Witala et al.



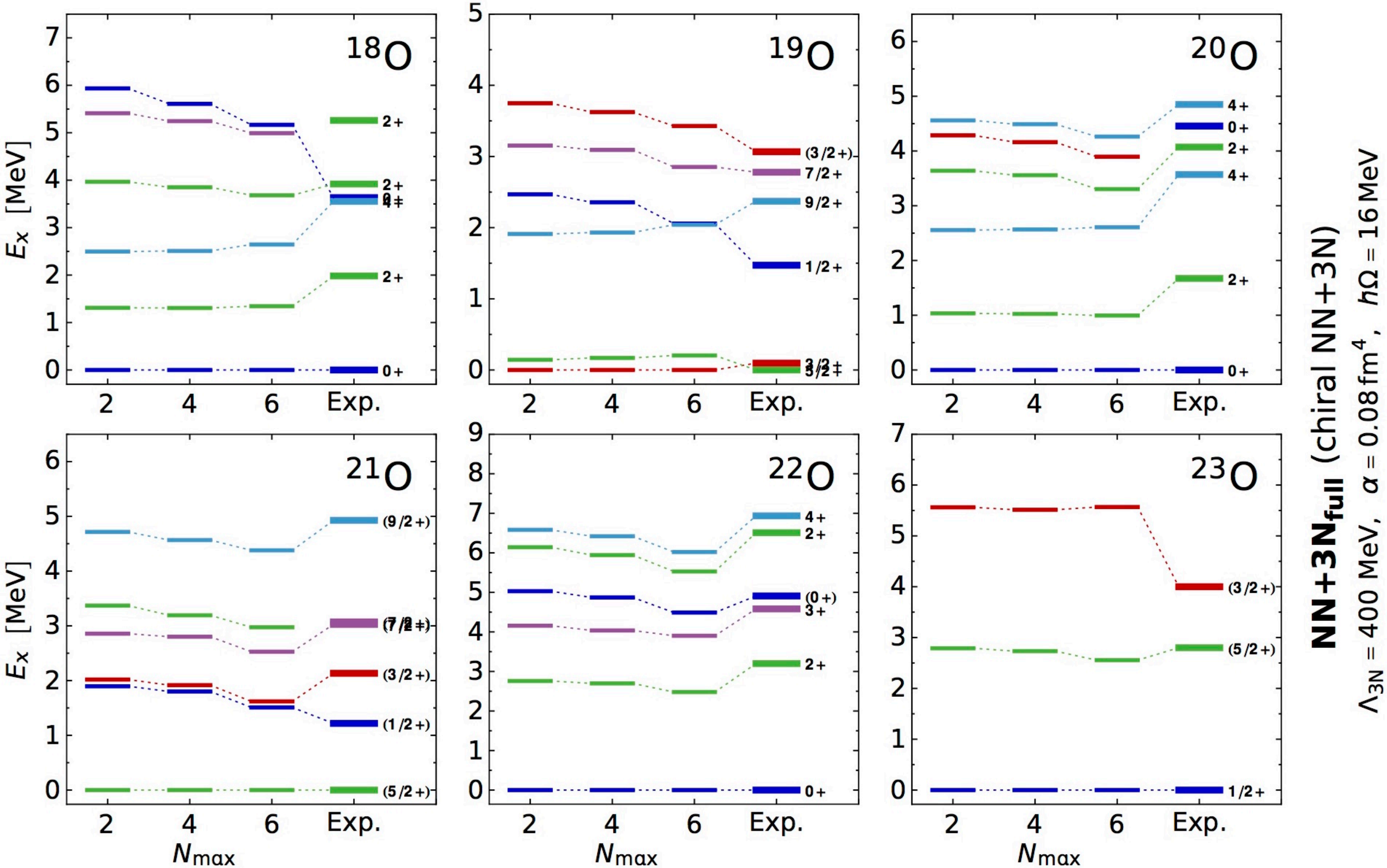
CURRENT STATUS AND OPEN ISSUES

- **Current status: 2NFs and 3NFs up to N3LO are applied in nuclear few- and many-body systems.**
- **In general, quite a bit of success, but some persistent problems remain.**
- **In the few-body sector: Ay puzzle, N-d break-up, ...**
- **Light nuclei: Spectra not perfect.**

SPECTRA OF SOME OXYGEN ISOTOPES

Hergert et al., PRL 110, 242501 (2013) & in prep.

From Roth



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PRL 117, 052501 (2016) PHYSICAL REVIEW LETTERS week ending 29 JULY 2016

Radii and Binding Energies in Oxygen Isotopes: A Challenge for Nuclear Forces

V. Lapoux,[1,*] V. Somà,[1] C. Barbieri,[2] H. Hergert,[3] J. D. Holt,[4] and S. R. Stroberg[4]
[1]CEA, Centre de Saclay, IRFU, Service de Physique Nucléaire, 91191 Gif-sur-Yvette, France
[2]Department of Physics, University of Surrey, Guildford GU2 7XH, United Kingdom
[3]National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA
[4]TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3
(Received 29 April 2016; published 27 July 2016)

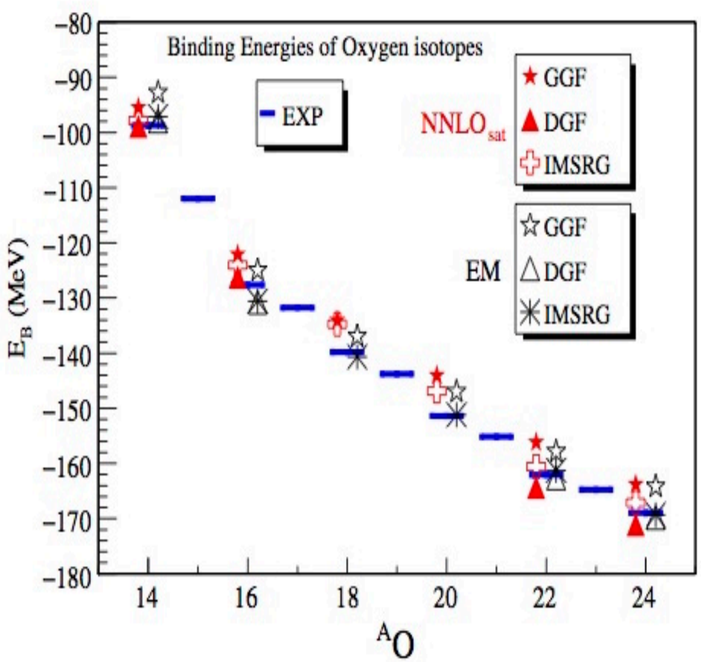

FIG. 1. Oxygen binding energies. Results from SCGF (DGF and GGF) and IMSRG calculations with EM and $\mathrm{NNLO_{sat}}$ are displayed along with experimental data.

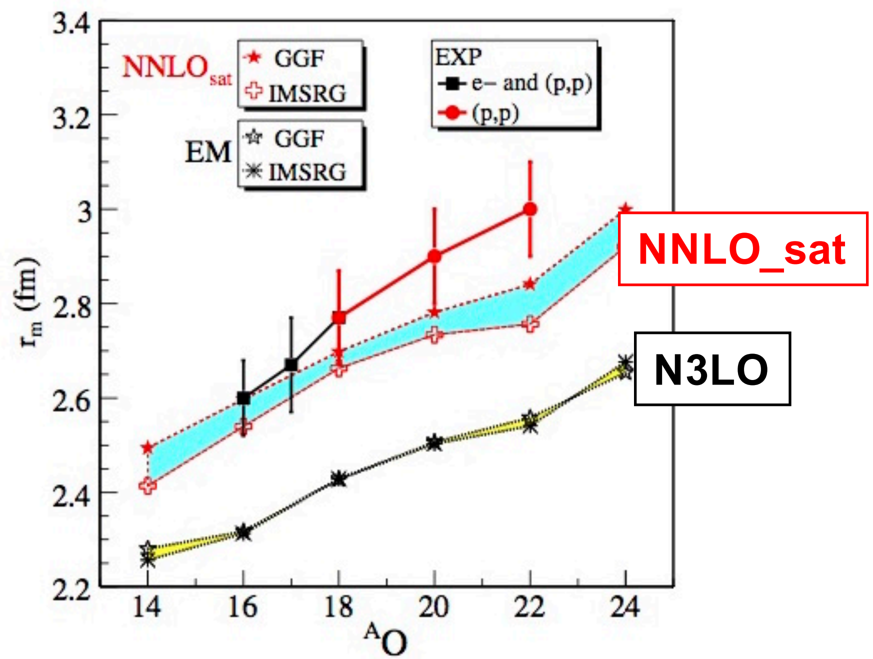

FIG. 5. Matter radii from our analysis and given in Table I, compared to calculations with EM [27–29] and $\mathrm{NNLO_{sat}}$ [36]. Bands span results from GGF and MR-IMSRG schemes.

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- **Overbinding of intermediate-mass nuclei**

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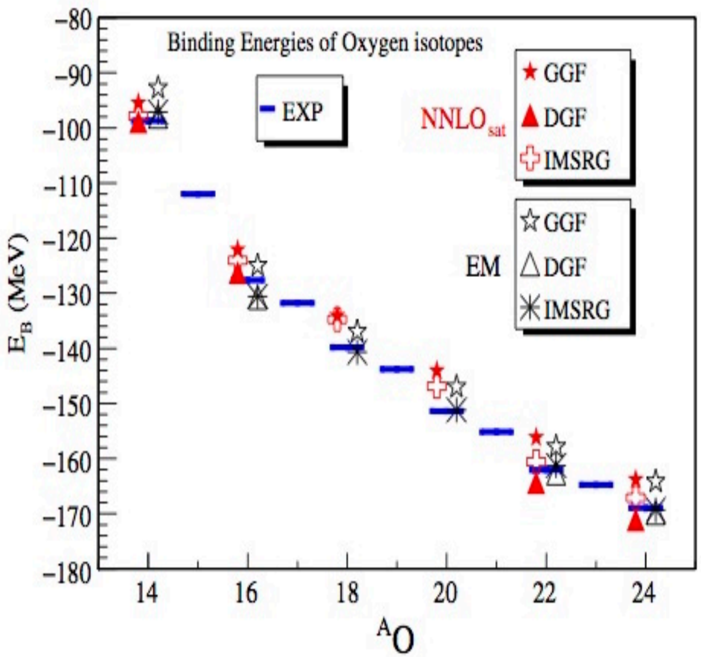

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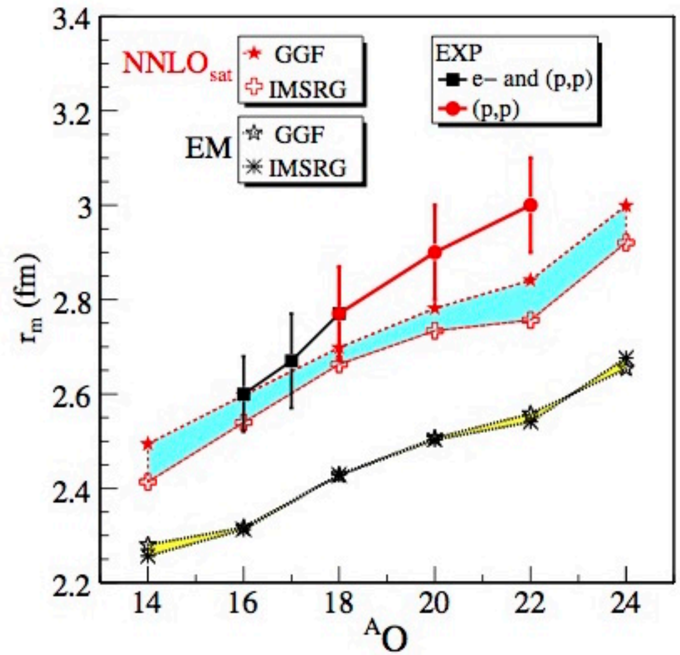

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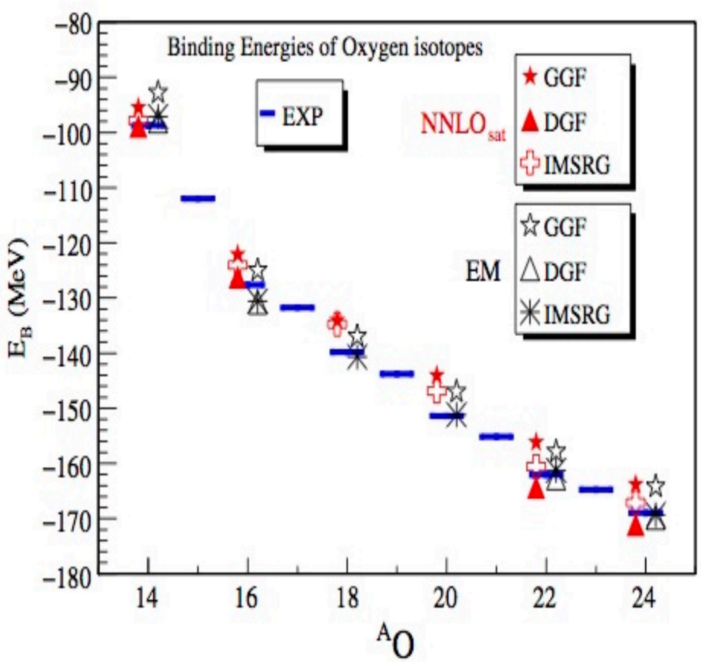

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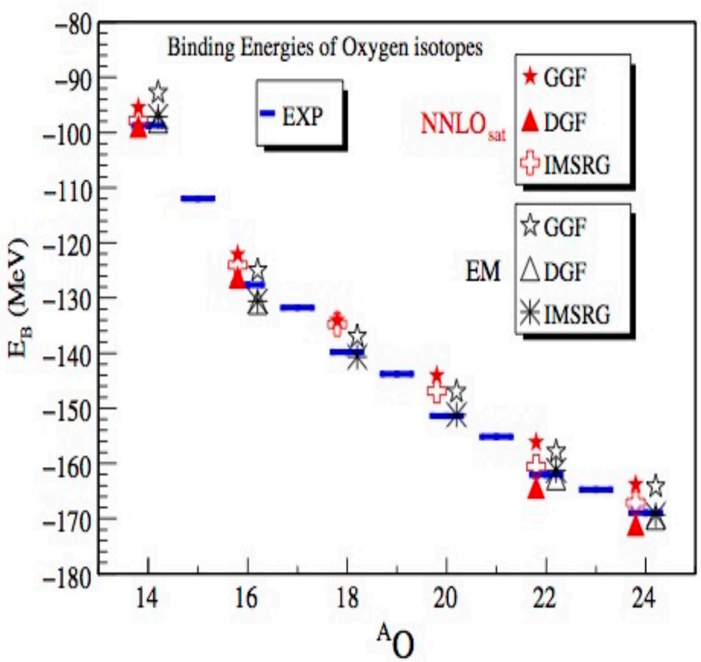
Overbinding of intermediate-mass nuclei

Oxygen



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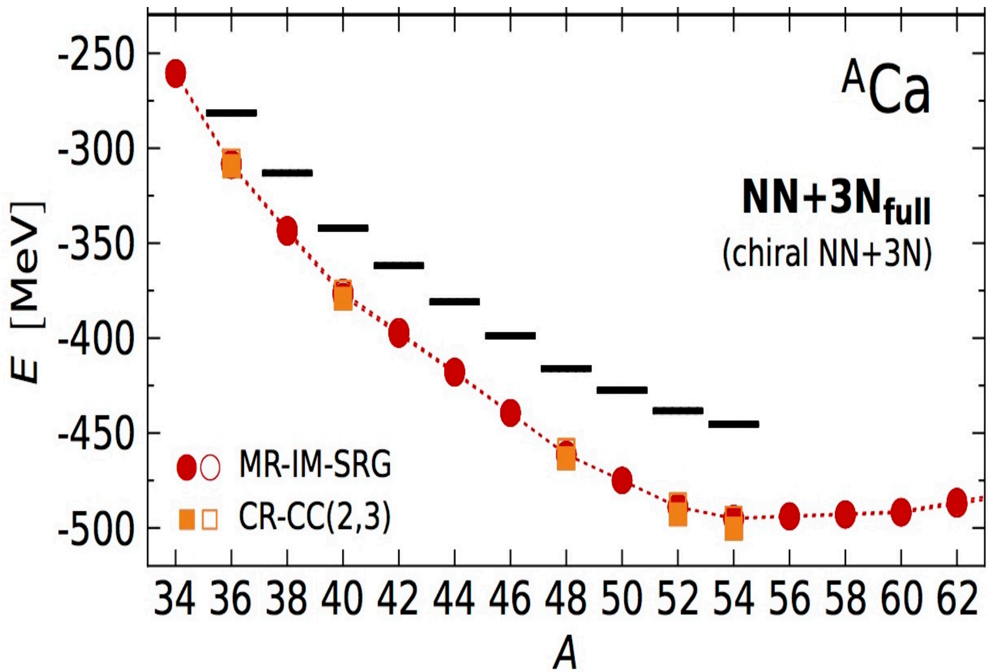
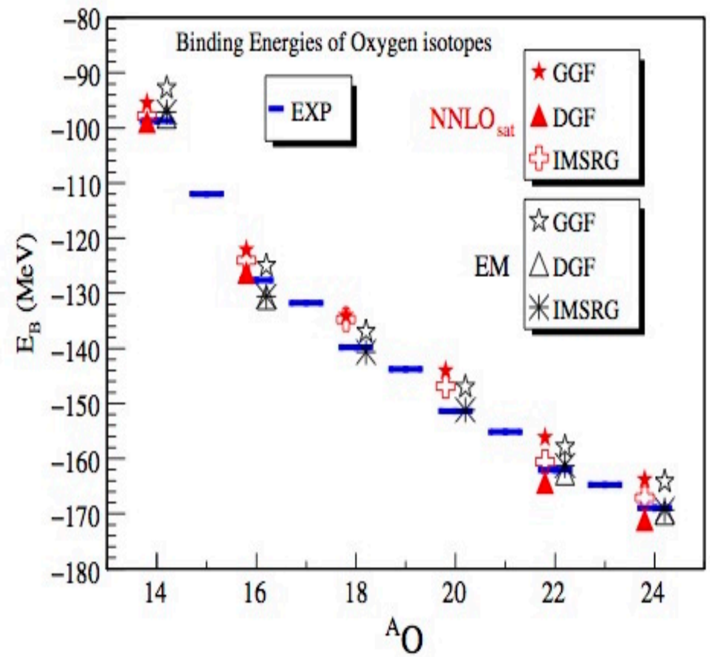
Calcium



From Hergert et al., PRC 90, 041302 (2014).

Overbinding of intermediate-mass nuclei

Oxygen



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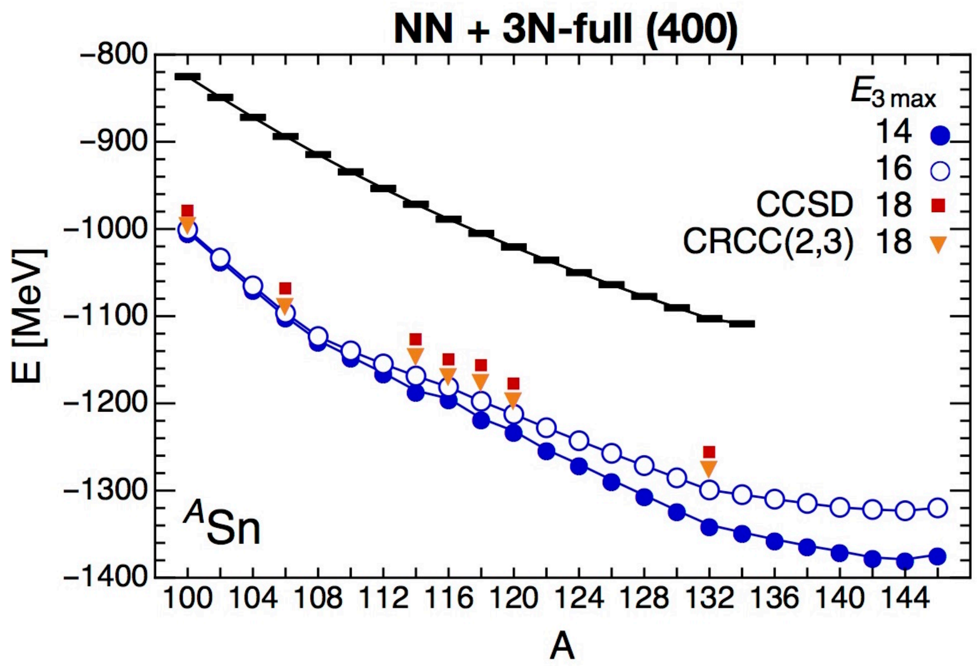
Tin



From Hergert

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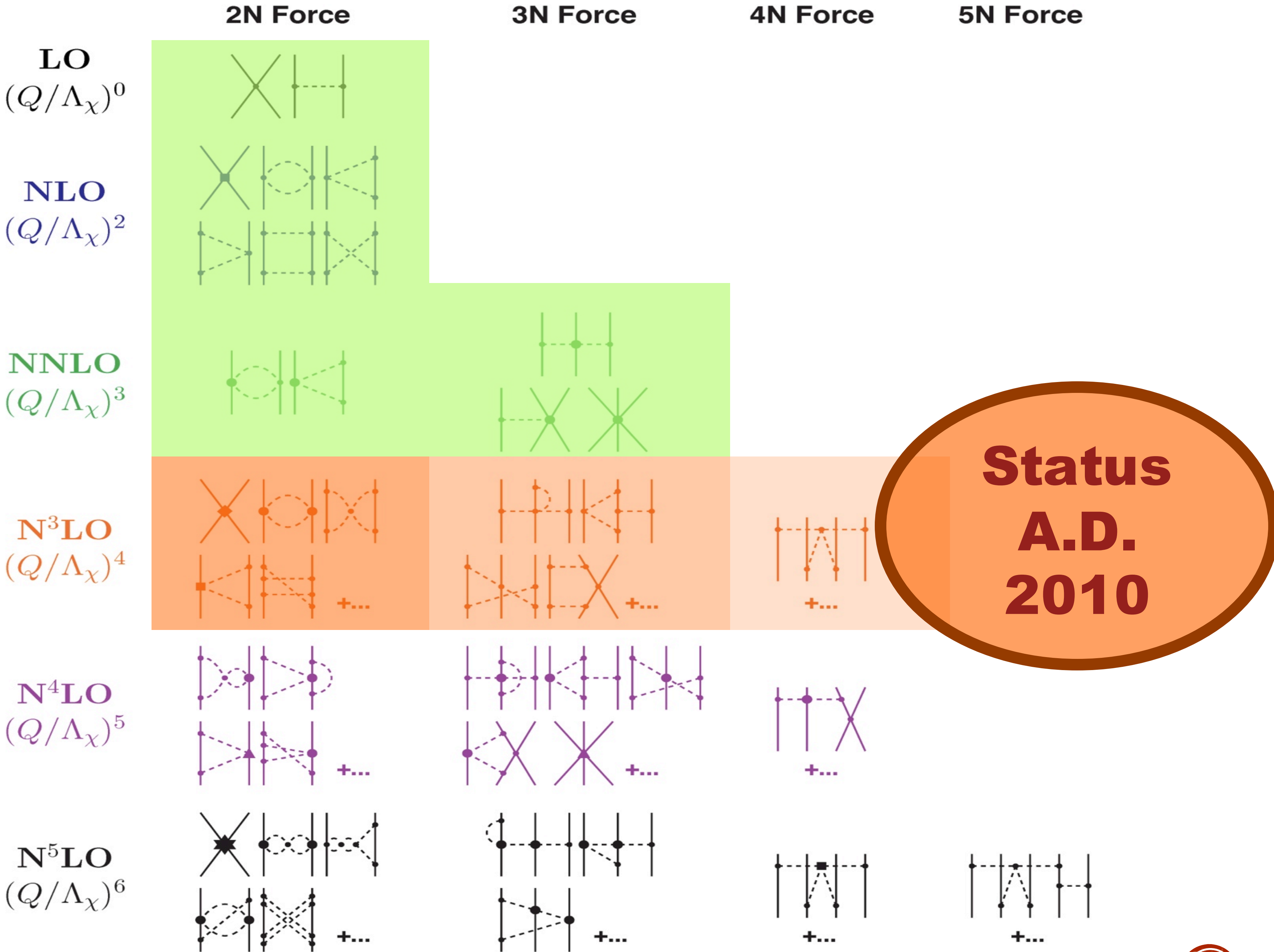
- **Light nuclei: Spectra not perfect.**
- **The radii of nuclei**
- **Overbinding of intermediate-mass nuclei**
- **Convergence of the chiral expansion in the many-body system**

BECAUSE OF THE PROBLEMS JUST POINTED OUT, IMPROVEMENT OF CURRENT NUCLEAR FORCES IS CALLED FOR.

- **How?**
- **Revisit the lower orders**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Status
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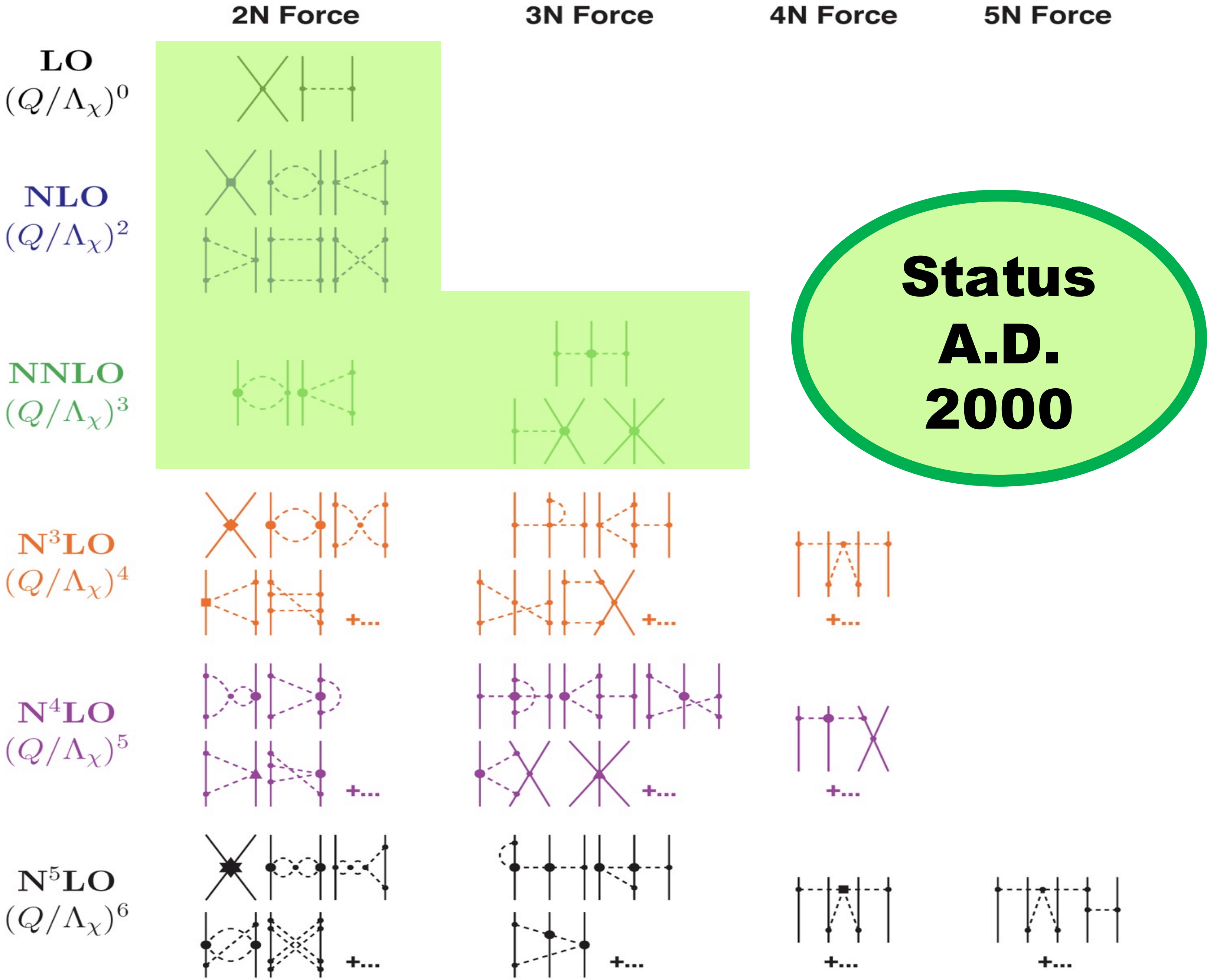

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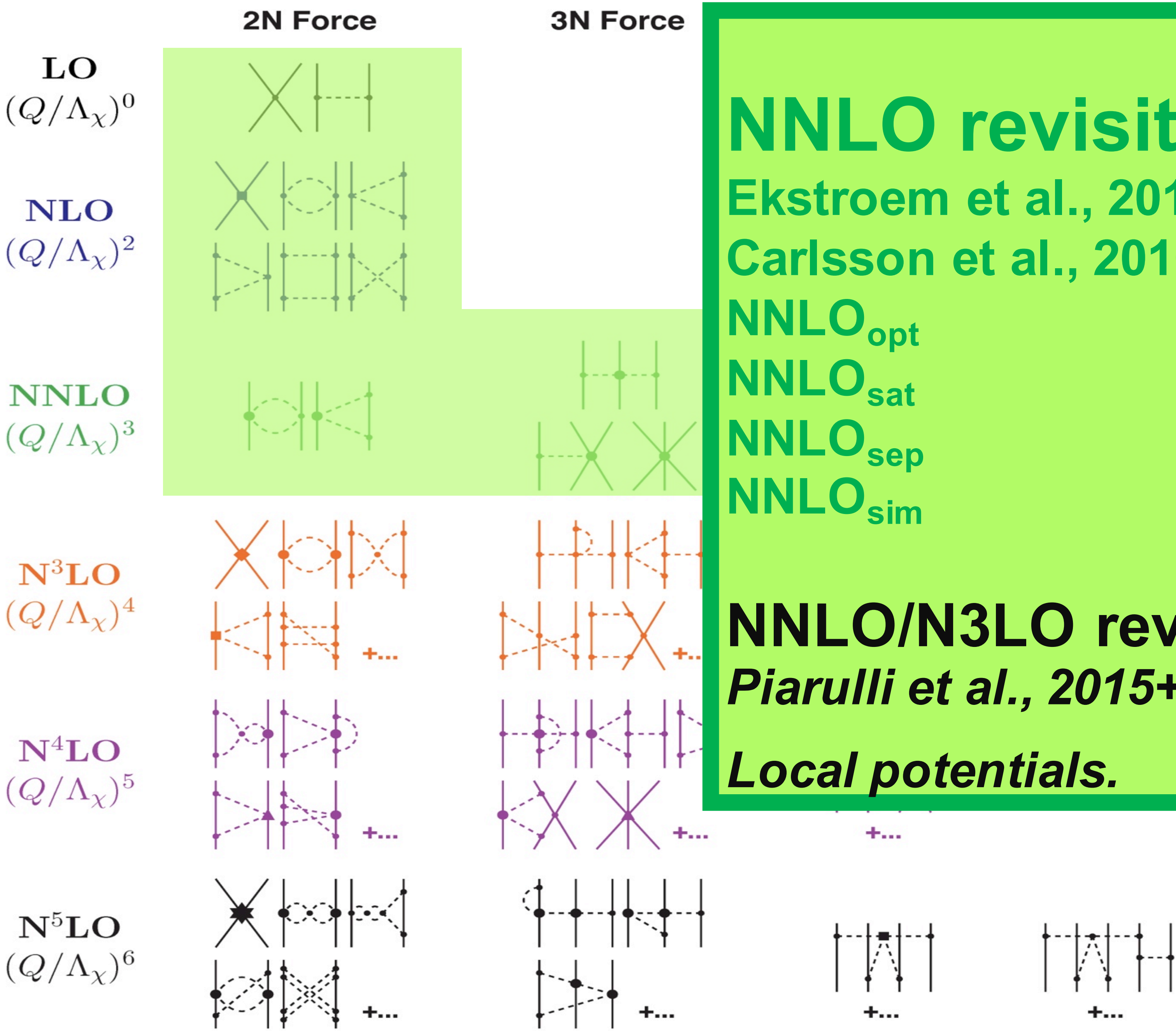


NNLO revisited:

Ekstroem et al., 2013+

Carlsson et al., 2016

NNLO$_{opt}$

NNLO$_{sat}$

NNLO$_{sep}$

NNLO$_{sim}$

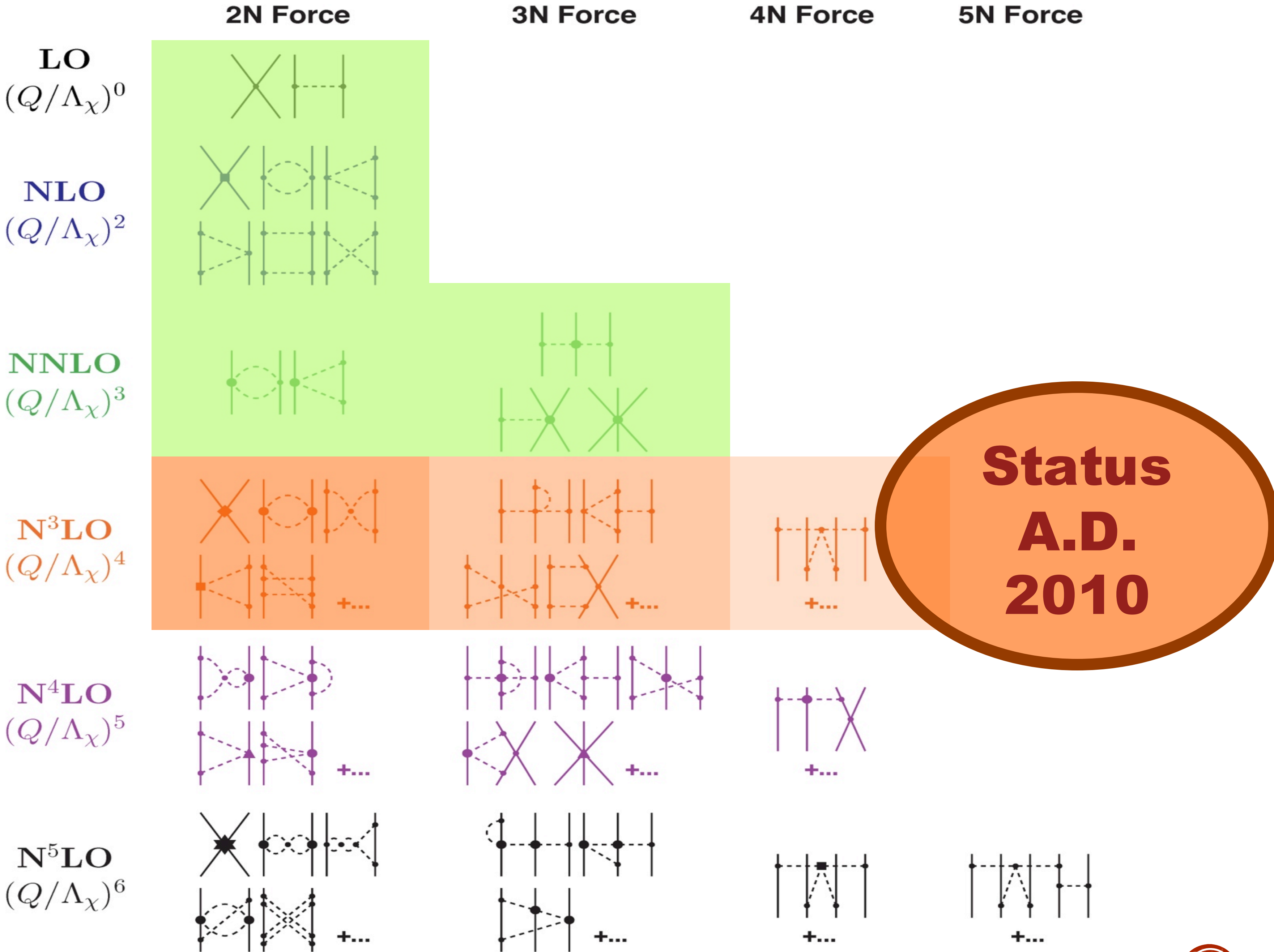
NNLO/N3LO revisited:

Piarulli et al., 2015+

Local potentials.

BECAUSE OF THE PROBLEMS JUST POINTED OUT, IMPROVEMENT OF CURRENT NUCLEAR FORCES IS CALLED FOR.

- **How?**
- **Revisit the lower orders**
- **Move on to higher orders**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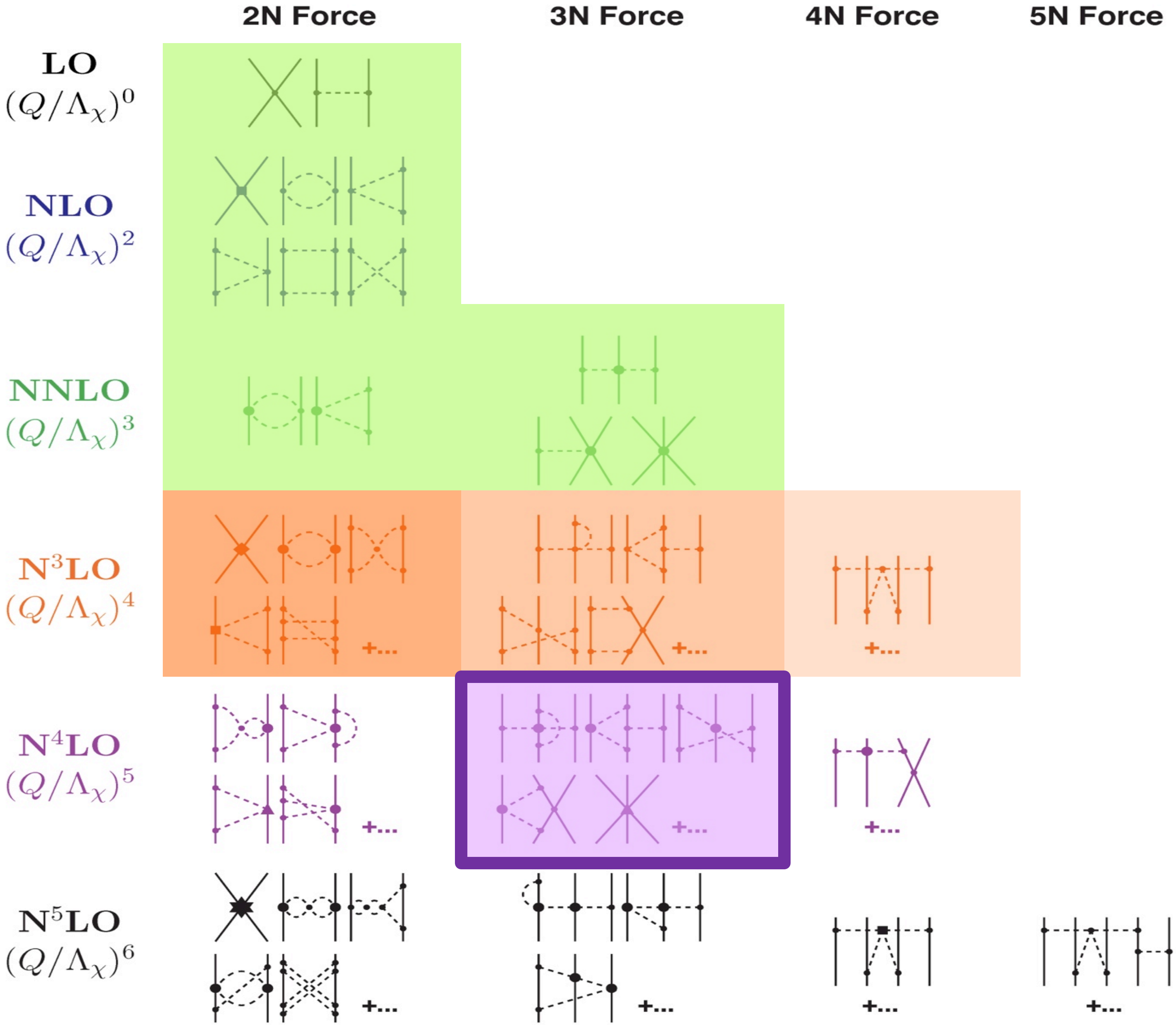
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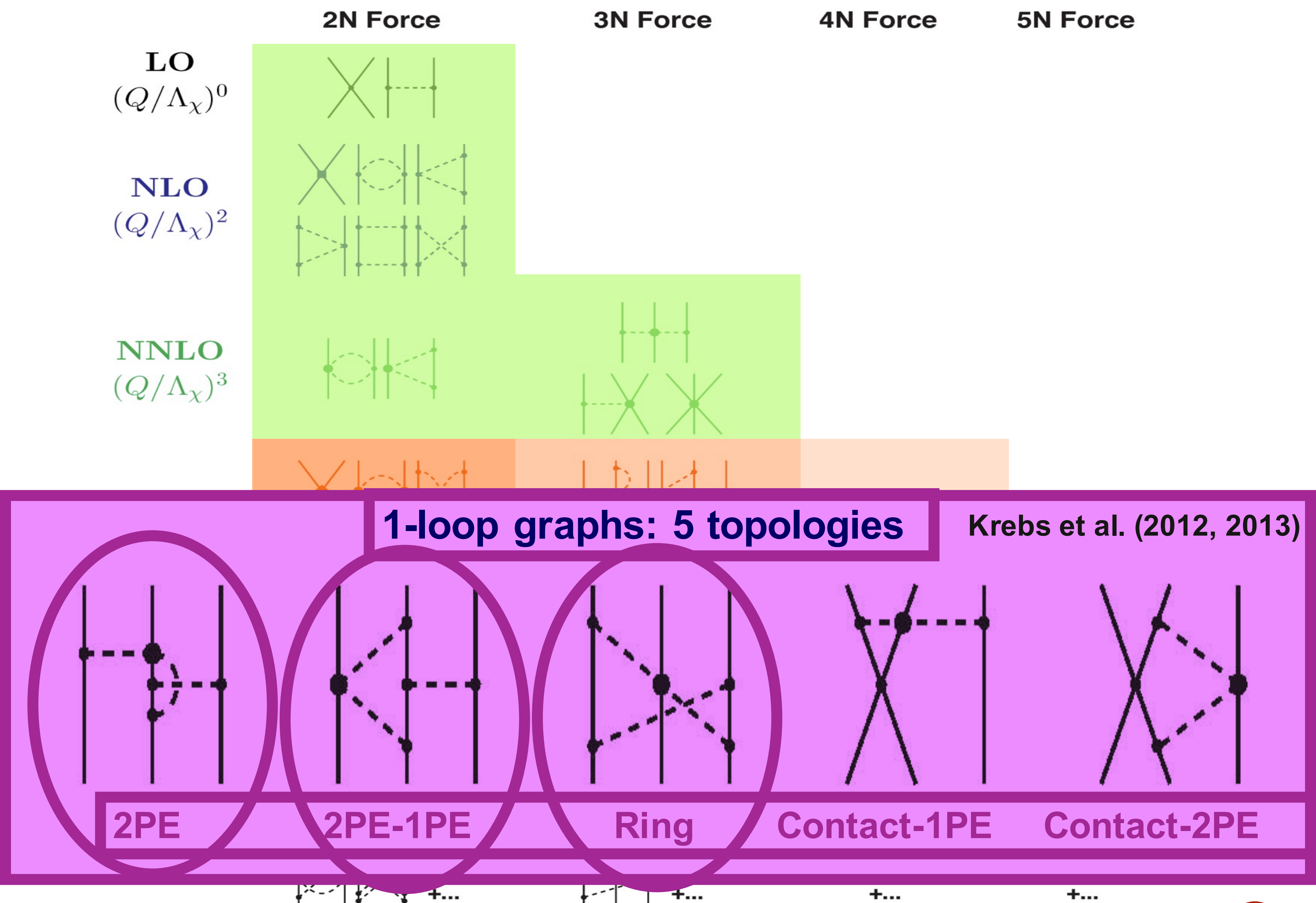
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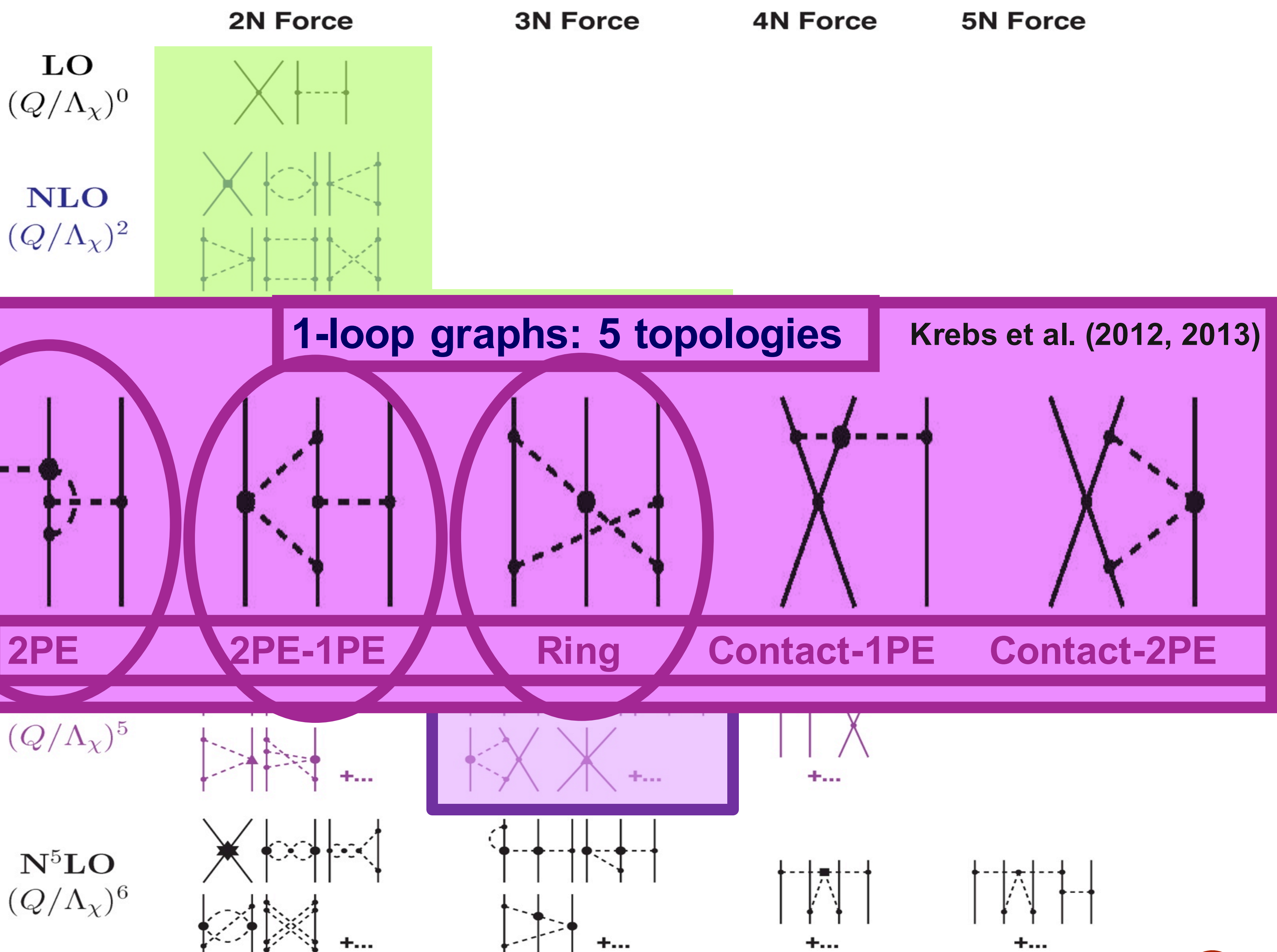
1-loop graphs: 5 topologies
Krebs et al. (2012, 2013)
2PE
2PE-1PE
Ring
Contact-1PE
Contact-2PE
+...
+...
+...
+...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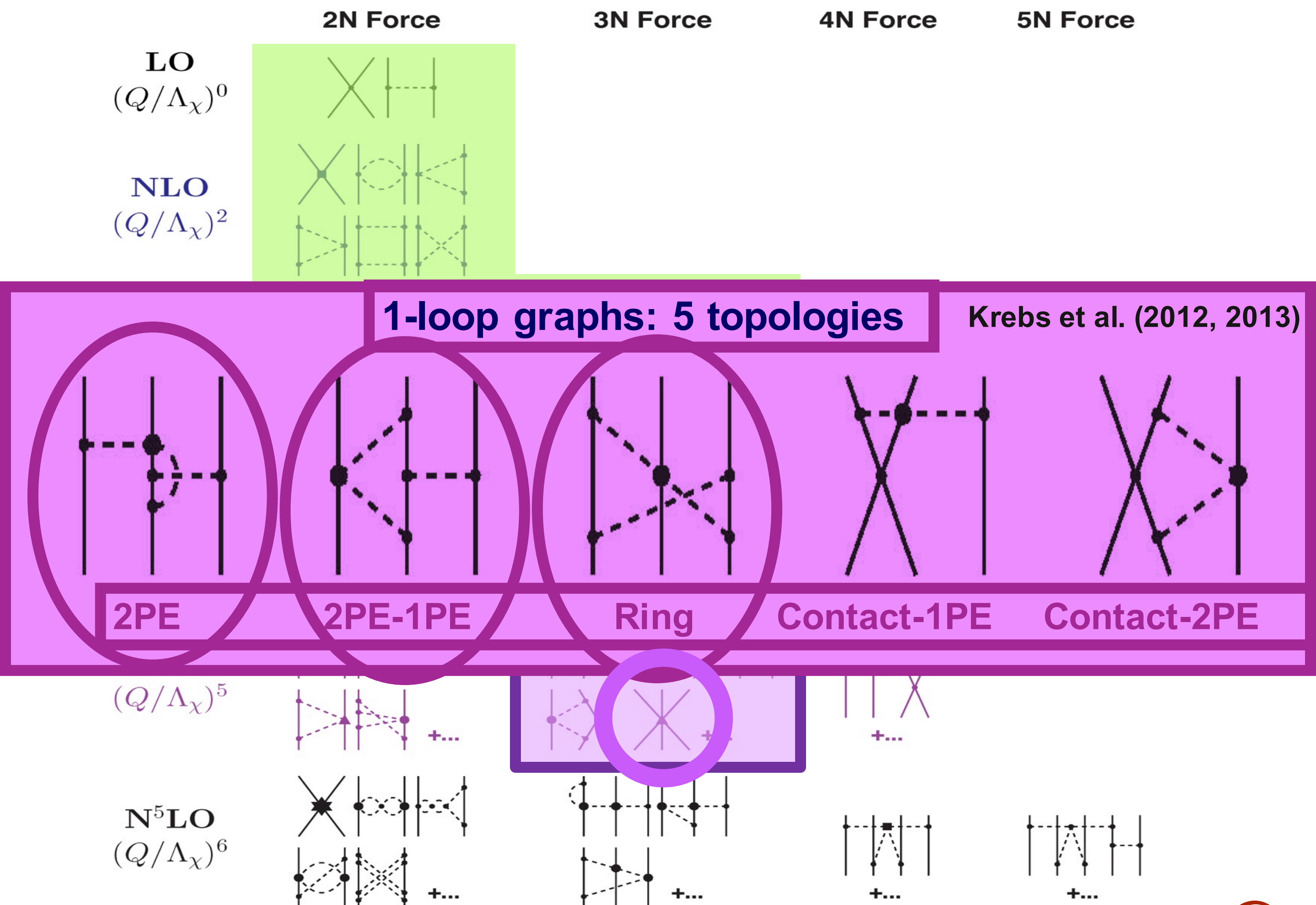

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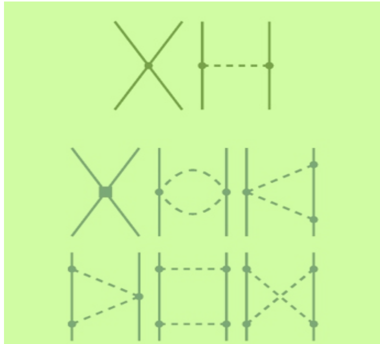
NLO $(Q/\Lambda_\chi)^2$



1-loop graphs: 5 topologies

Krebs et al. (2012, 2013)

PE

3NF contacts at N4LO

Girlanda, Kievsky, Viviani, PRC 84, 014001 (2011)

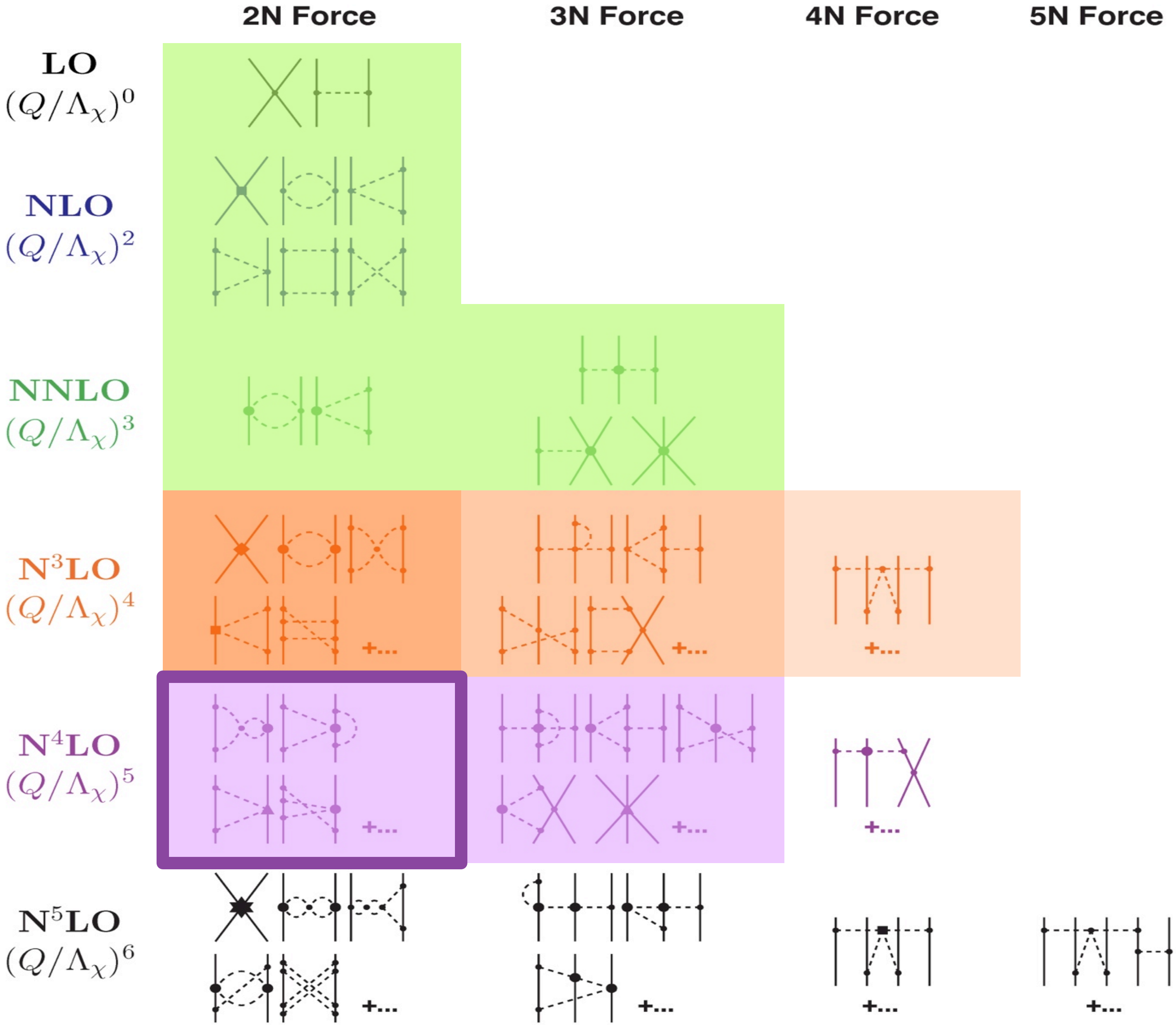
$\mathbf{k}_i = \mathbf{p}_i - \mathbf{p}'_i$ and $\mathbf{Q}_i = \mathbf{p}_i + \mathbf{p}'_i$, $\mathbf{p}_i$ and $\mathbf{p}'_i$ being the initial and final momenta of nucleon i, the potential in momentum space is found to be

$$\begin{aligned} V = \sum_{i\neq j\neq k} \Big[& -E_1\mathbf{k}_i^2 - E_2\mathbf{k}_i^2\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j - E_3\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j - E_4\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \\ & -E_5\left(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2\right) - E_6\left(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2\right))\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \\ & +\frac{i}{2}E_7\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j) + \frac{i}{2}E_8\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k \\ & -E_9\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_j\cdot\boldsymbol{\sigma}_j - E_{10}\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_j\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\Big], \end{aligned} \tag{15}$$

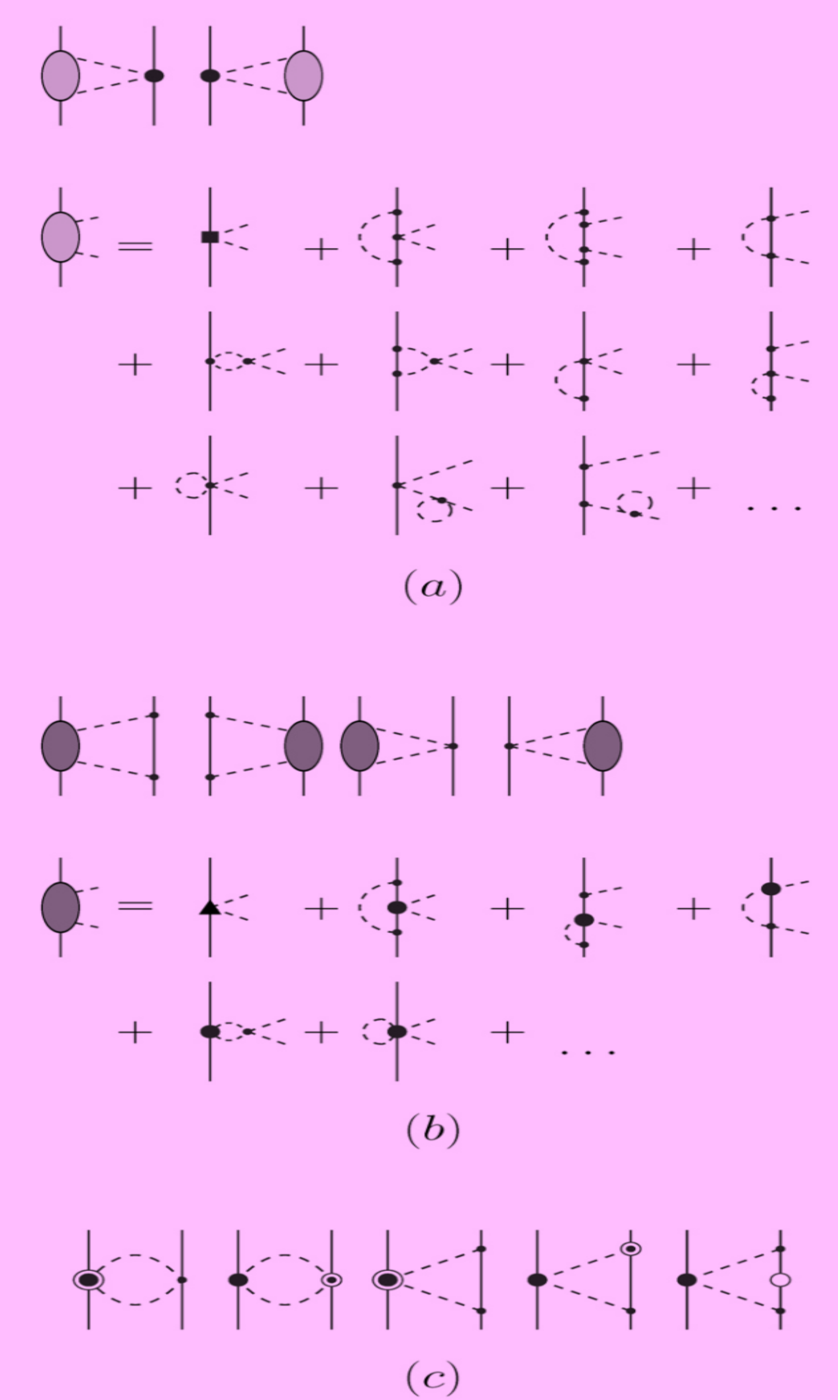
All possible 20 isospin-spin-momentum/position structures occur in the 3NF at N4LO!

Epelbaum et al., Eur. Phys. J. A51, 26 (2015)

Generators $\mathcal{G}$ in momentum space	Generators $\tilde{\mathcal{G}}$ in coordinate space
$\mathcal{G}_1 = 1$	$\tilde{\mathcal{G}}_1 = 1$
$\mathcal{G}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$	$\tilde{\mathcal{G}}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$
$\mathcal{G}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_2$
$\mathcal{G}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$	$\tilde{\mathcal{G}}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$
$\mathcal{G}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_8 = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_8 = \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_3$
$\mathcal{G}_9 = \vec{q}_1 \cdot \vec{\sigma}_3 \vec{q}_3 \cdot \vec{\sigma}_1$	$\tilde{\mathcal{G}}_9 = \hat{r}_{23} \cdot \vec{\sigma}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1$
$\mathcal{G}_{10} = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{10} = \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_2 \cdot \vec{\sigma}_1 \vec{q}_2 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{13} \cdot \vec{\sigma}_1\, \hat{r}_{13} \cdot \vec{\sigma}_3$
$\mathcal{G}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_2\, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{\sigma}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_3 \cdot \vec{q}_1 \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$	$\tilde{\mathcal{G}}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_3 \cdot \hat{r}_{23}\, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$
$\mathcal{G}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{12}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$

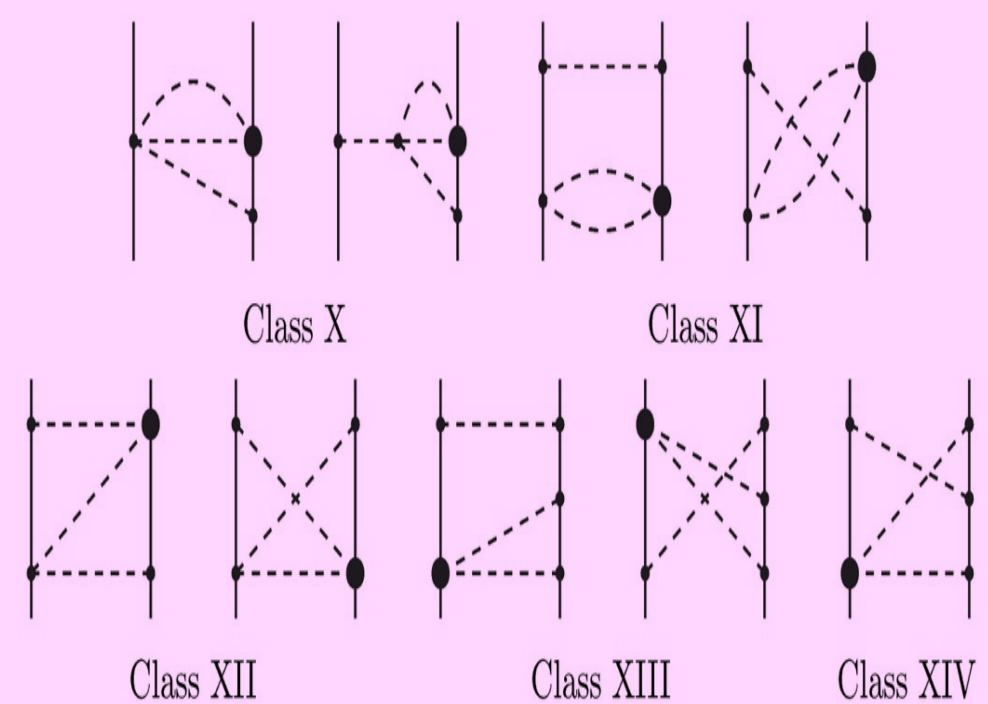


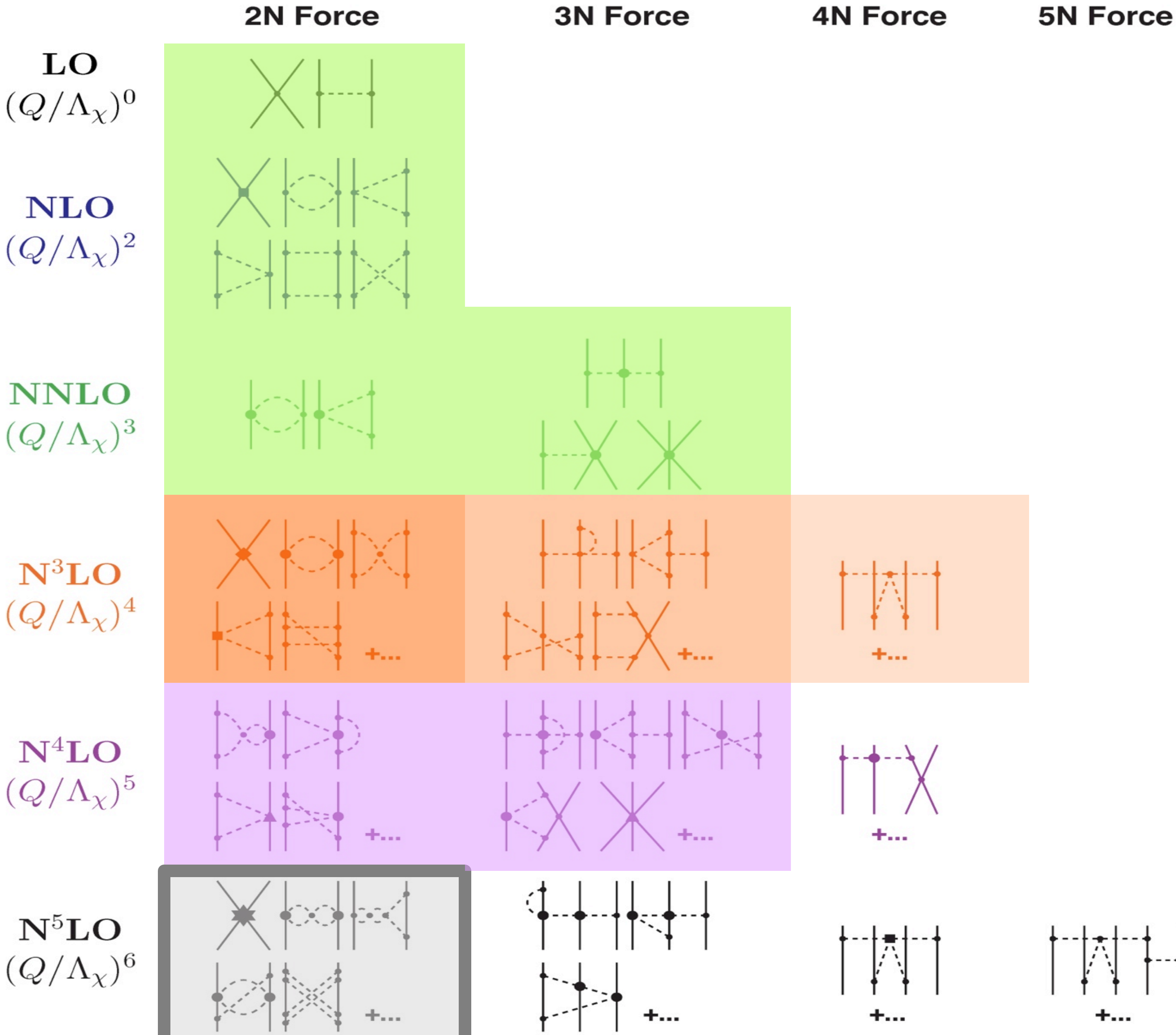
2N Force
3N Force
4N Force
5N Force
LO
$(Q/\Lambda_\chi)^0$
NLO
$(Q/\Lambda_\chi)^2$
NNLO
$(Q/\Lambda_\chi)^3$
N^{3}LO
$(Q/\Lambda_\chi)^4$
N^{4}LO
$(Q/\Lambda_\chi)^5$
N^{5}LO
$(Q/\Lambda_\chi)^6$
+...



N4LO 2NF Contributions

Entem, Kaiser, Machleidt, Nosyk,
PRC 91, 014002 (2015)




2N Force
3N Force
4N Force
5N Force
LO
$(Q/\Lambda_\chi)^0$
NLO
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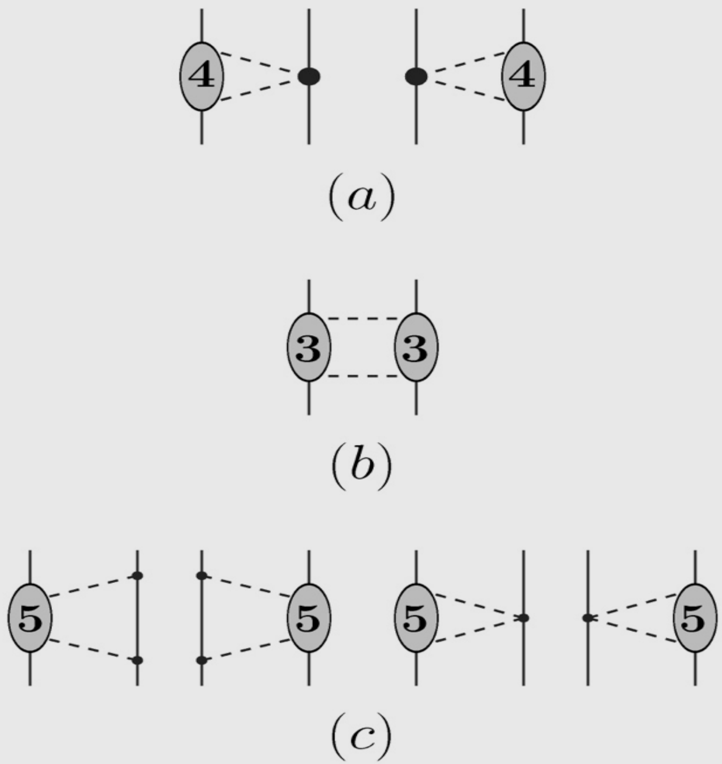
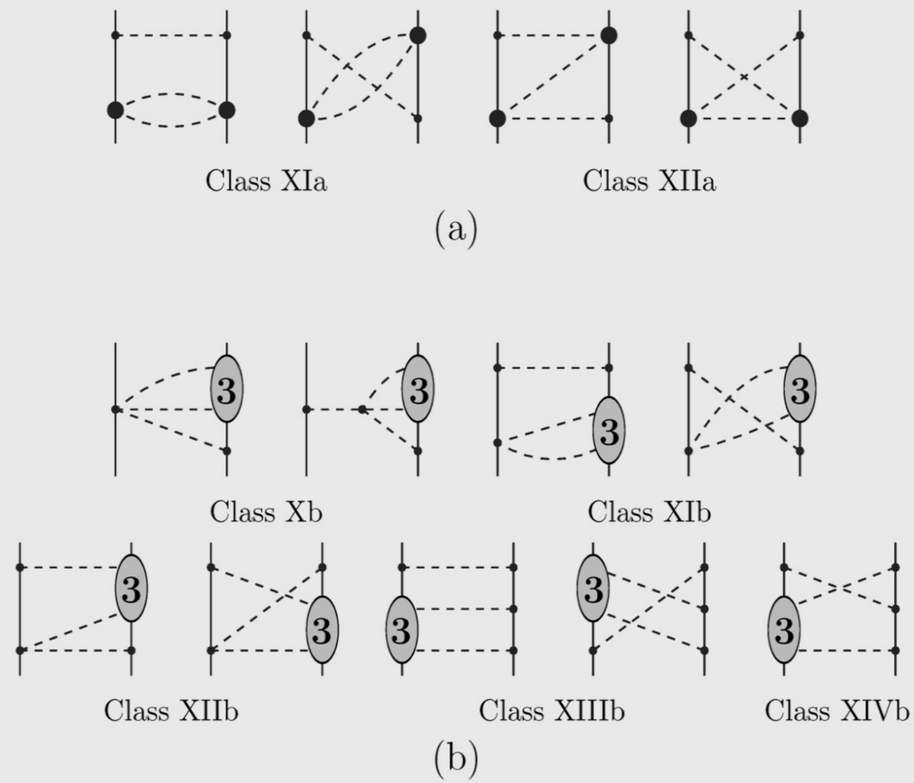
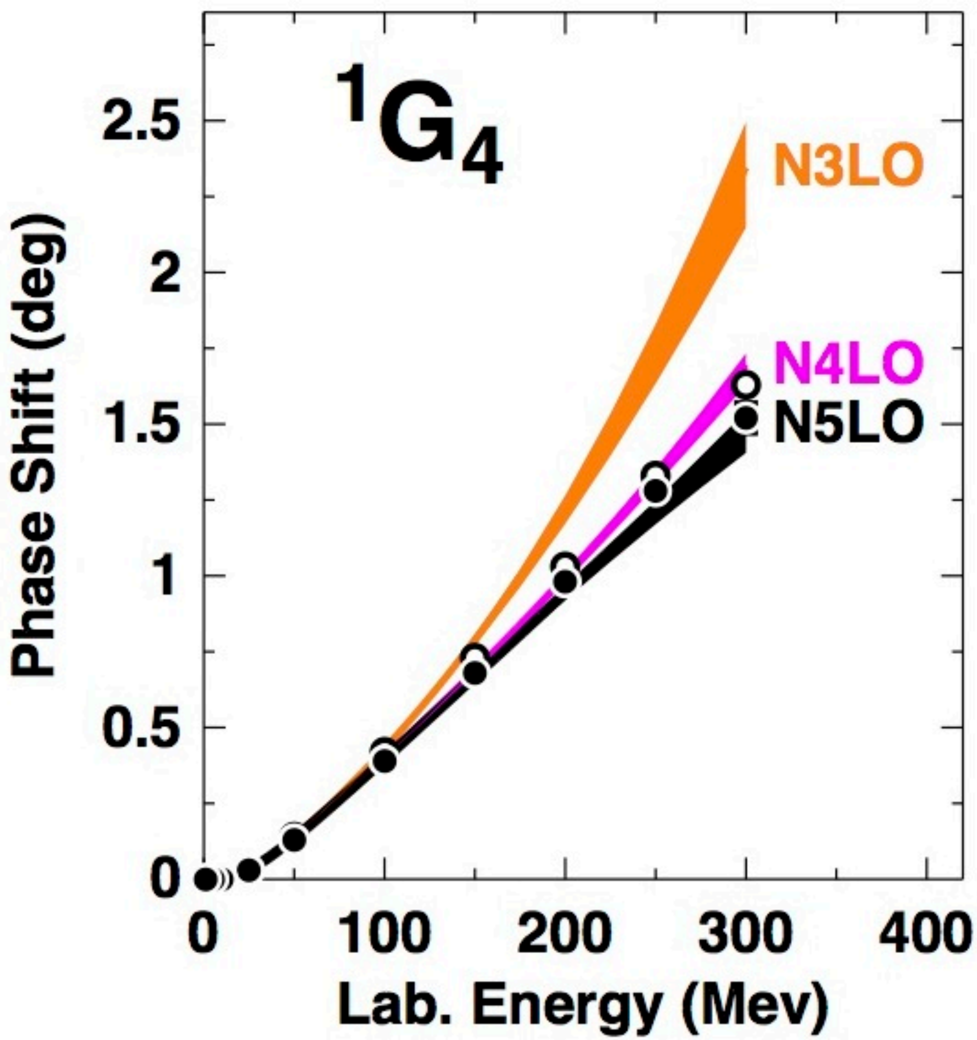
NNLO
$(Q/\Lambda_\chi)^3$
N^{3}LO
$(Q/\Lambda_\chi)^4$
N^{4}LO
$(Q/\Lambda_\chi)^5$
N^{5}LO
$(Q/\Lambda_\chi)^6$
+...

N5LO 2NF Contributions

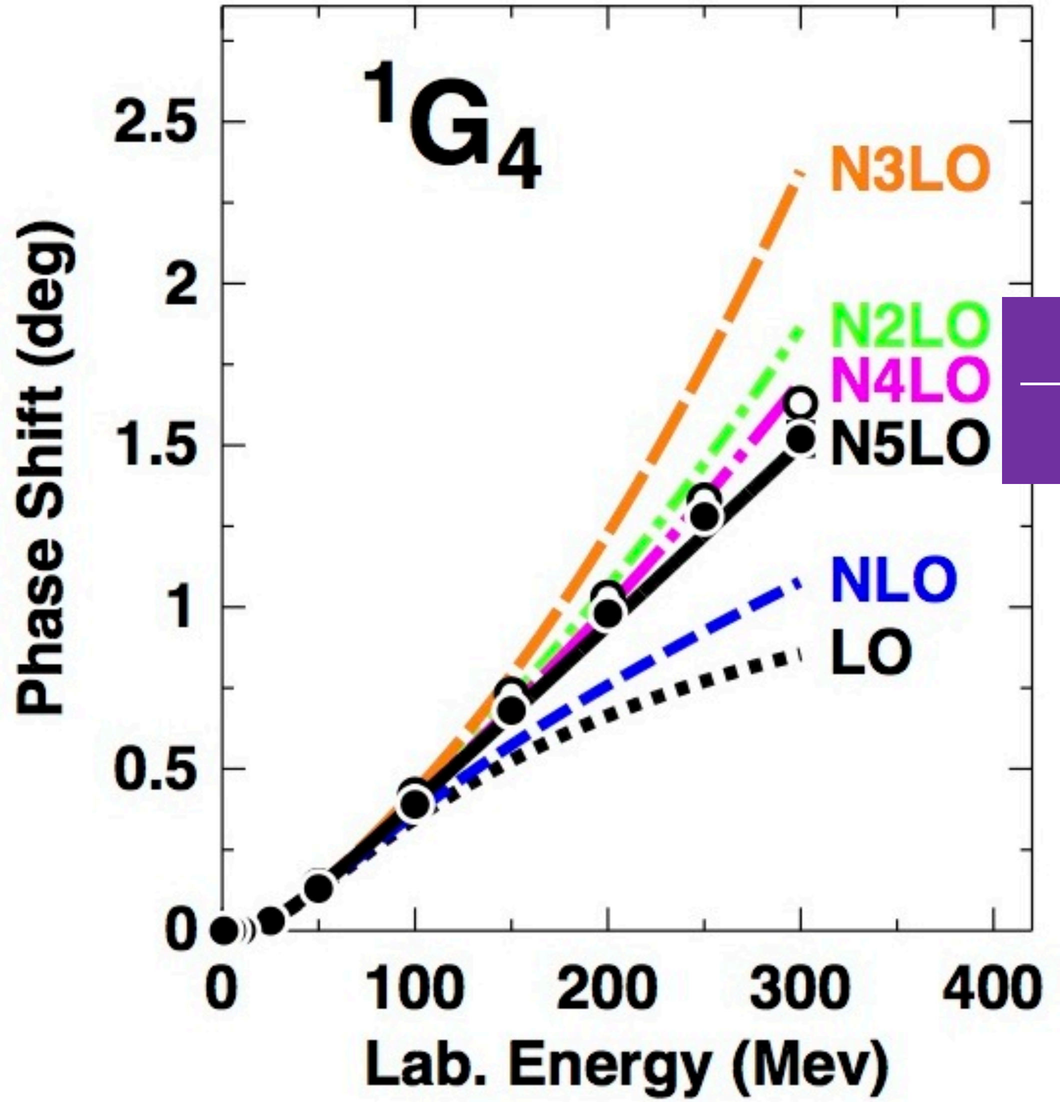
Entem, Kaiser, Machleidt, Nosyk,
PRC 92, 064001 (2015)





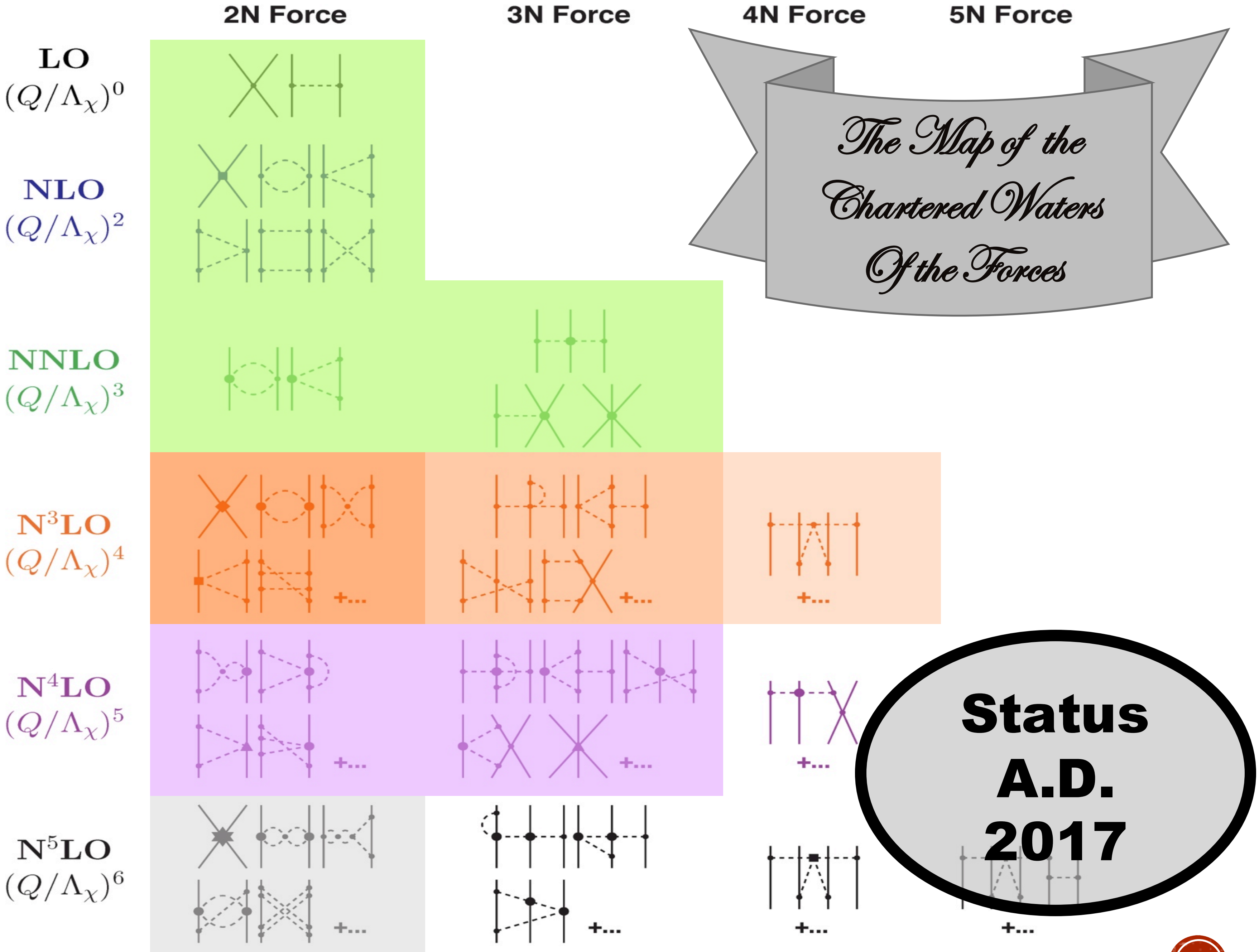


From Entem, Kaiser, Machleidt, Nosyk, PRC 91, 014002 (2015)



Converged at N4LO

From Entem, Kaiser, Machleidt, Nosyk, PRC 92, 064001 (2015)


2N Force
3N Force
4N Force
5N Force
The Map of the Chartered Waters Of the Forces
LO
$(Q/\Lambda_\chi)^0$
NLO
$(Q/\Lambda_\chi)^2$
NNLO
$(Q/\Lambda_\chi)^3$
N^{3}LO
$(Q/\Lambda_\chi)^4$
N^{4}LO
$(Q/\Lambda_\chi)^5$
N^{5}LO
$(Q/\Lambda_\chi)^6$
+...
Status A.D. 2017

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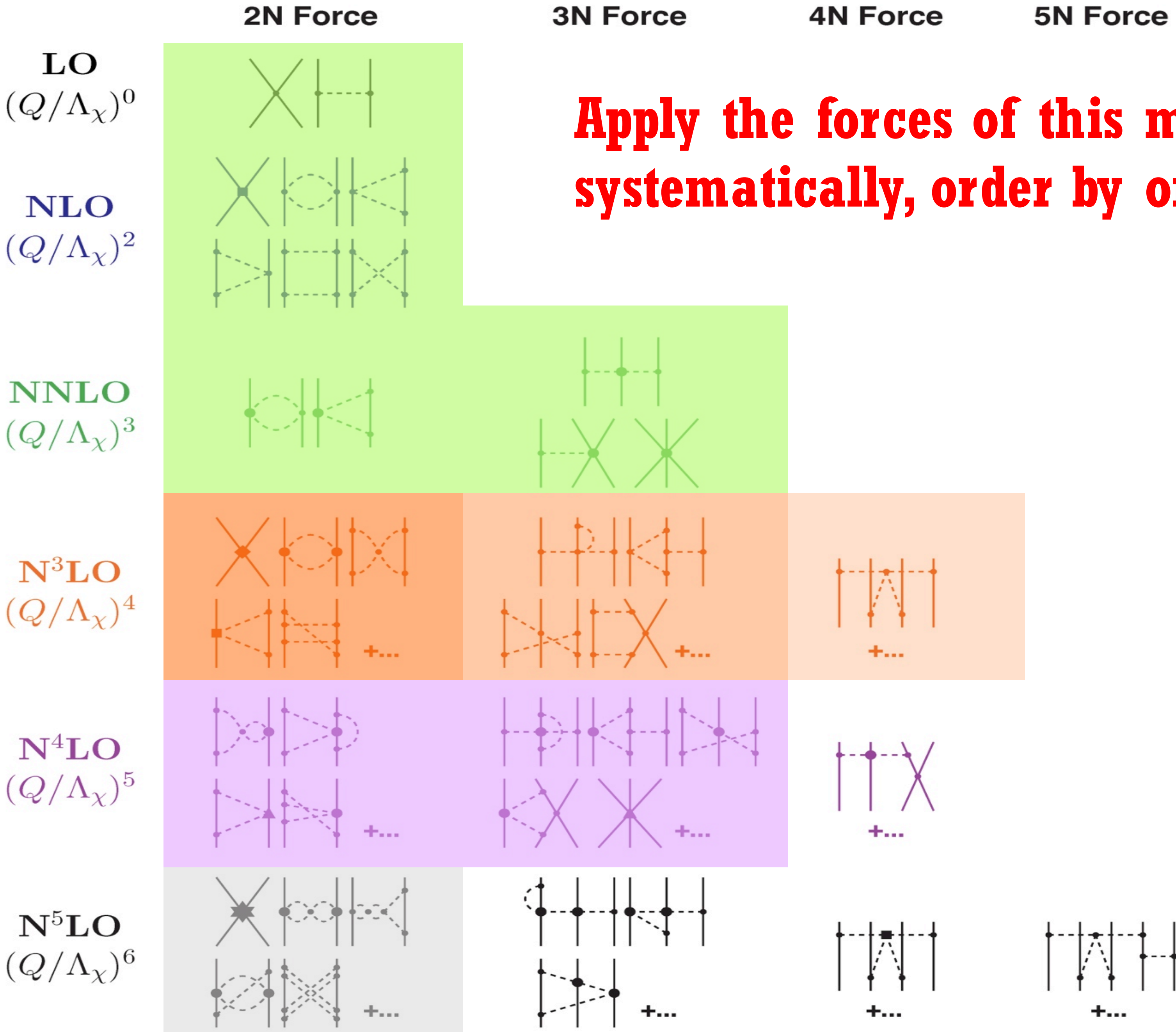
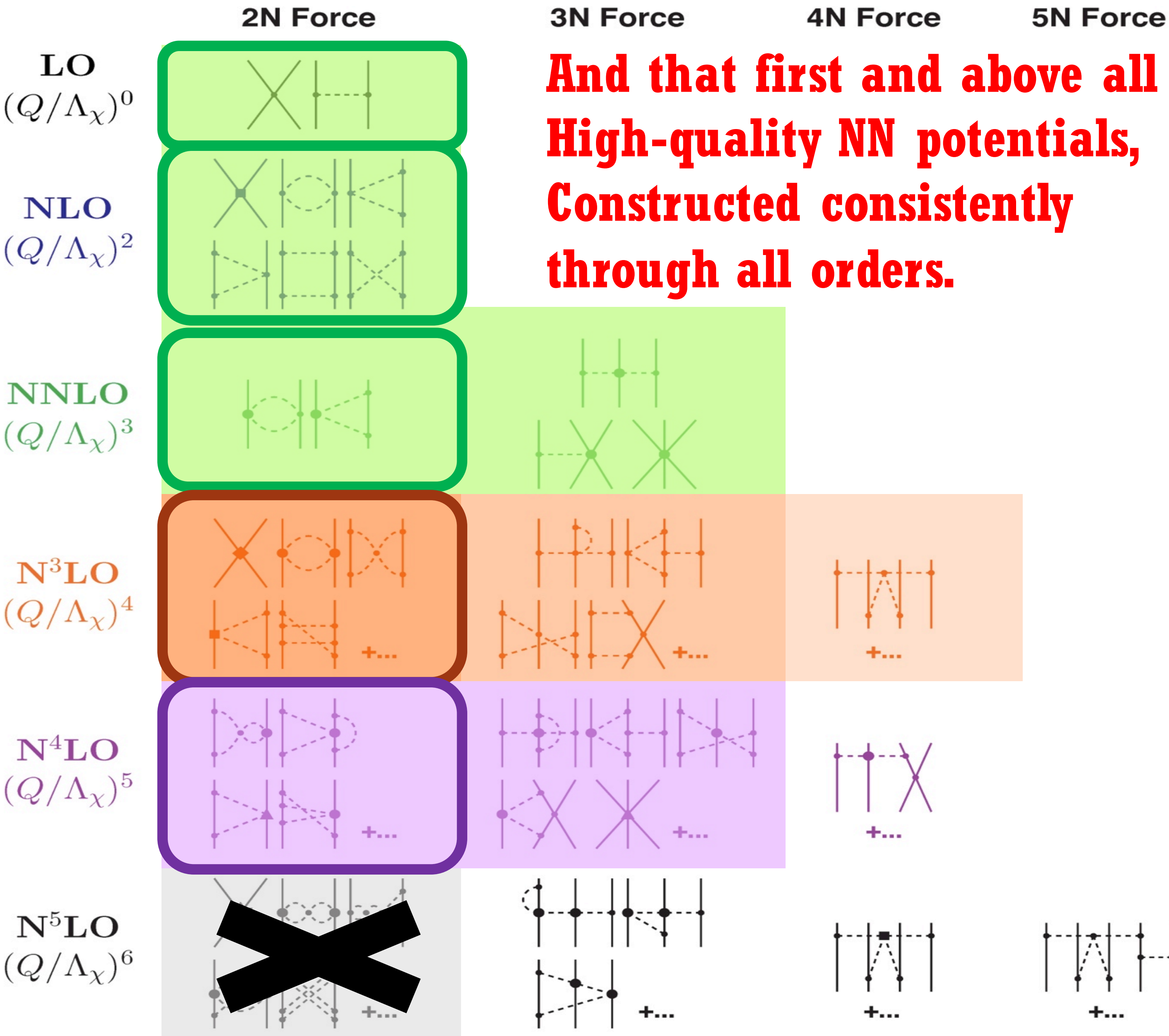
NOW THAT WE HAVE CHARTERED THE WATERS OF THE FORCES, HOW DO WE ADDRESS THE ISSUES?



Apply the forces of this map systematically, order by order.

And that first and above all requires High-quality NN potentials, Constructed consistently through all orders.



"HIGH QUALITY","CONSISTENTLY", … WHAT DOES THAT MEAN?

- **Use π-N LECs determined in π-N analysis with the highest possible precision: Roy-Steiner Analysis (Hoferichter et al., PRL 115, 192301 (2015)).**

PRL 115, 192301 (2015) PHYSICAL REVIEW LETTERS week ending 6 NOVEMBER 2015

Matching Pion-Nucleon Roy-Steiner Equations to Chiral Perturbation Theory

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(Received 28 July 2015; published 4 November 2015)

We match the results for the subthreshold parameters of pion-nucleon scattering obtained from a solution of Roy-Steiner equations to chiral perturbation theory up to next-to-next-to-next-to-leading order, to extract the pertinent low-energy constants including a comprehensive analysis of systematic uncertainties and correlations. We study the convergence of the chiral series by investigating the chiral expansion of threshold parameters up to the same order and discuss the role of the $\Delta(1232)$ resonance in this context. Results for the low-energy constants are also presented in the counting scheme usually applied in chiral nuclear effective field theory, where they serve as crucial input to determine the long-range part of the nucleon-nucleon potential as well as three-nucleon forces.

*2015 Klaus Erkelenz Prize Winners (University of Bonn, Germany)

PRL 115, 192301 (2015) PHYSICAL REVIEW LETTERS week ending 6 NOVEMBER 2015

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MAIN CHARACTERISTICS:

- **Set of coupled partial-wave dispersion relations constraint by analyticity, unitarity, and crossing symmetry.**
- **Additional crucial constraint: High-accuracy π-N scattering lengths extracted from pionic atoms.**
- **Matching to π-N LECs done in the subthreshold region, which is best for nuclear forces.**
- **Comprehensive error analysis.**
- **Small errors.**

π-N LECs from Roy–Steiner Analysis

(Hoferichter et al., PRL 115, 192301 (2015))

TABLE II: The πN LECs as determined in the Roy-Steiner-equation analysis of πN scattering conducted in Ref. [35]. The given orders of the chiral expansion refer to the NN system. Note that the orders, at which the LECs are extracted from the πN system, are always lower by one order as compared of the NN system in which the LECs are applied. The c_i, $\bar{d}_i$, and $\bar{e}_i$ are the LECs of the second, third, and fourth order πN Lagrangian [26] and are in units of GeV^{-1}, GeV^{-2}, and GeV^{-3}, respectively. The uncertainties in the last digits are given in parentheses after the values.

	NNLO	N^{3}LO	N^{4}LO
c_1	−0.74(2)	−1.07(2)	−1.10(3)
c_2	—	3.20(3)	3.57(4)
c_3	−3.61(5)	−5.32(5)	−5.54(6)
c_4	2.44(3)	3.56(3)	4.17(4)
$\bar{d}_1 + \bar{d}_2$	—	1.04(6)	6.18(8)
$\bar{d}_3$	—	−0.48(2)	−8.91(9)
$\bar{d}_5$	—	0.14(5)	0.86(5)
$\bar{d}_{14} - \bar{d}_{15}$	—	−1.90(6)	−12.18(12)
$\bar{e}_{14}$	—	—	1.18(4)
$\bar{e}_{17}$	—	—	−0.18(6)

Very small errors!

RECALL A TYPICAL PROBLEM FROM THE PAST …

- One had to assume that, e.g., $c_3 \cong 3.4 - 6.0$
- Leading to a huge uncertainty for the 3NF contribution.
- Inconsistency with c_3 used in the NN interaction.
- **This is all over now!**
- Uncertainty of the NN interaction due to the uncertainty in c_i's absolutely negligible.
- Uncertainty of the 3NF contribution due to the uncertainty in c_i's : negligible as compared to truncation error.

"HIGH QUALITY","CONSISTENTLY", … WHAT DOES THAT MEANS?

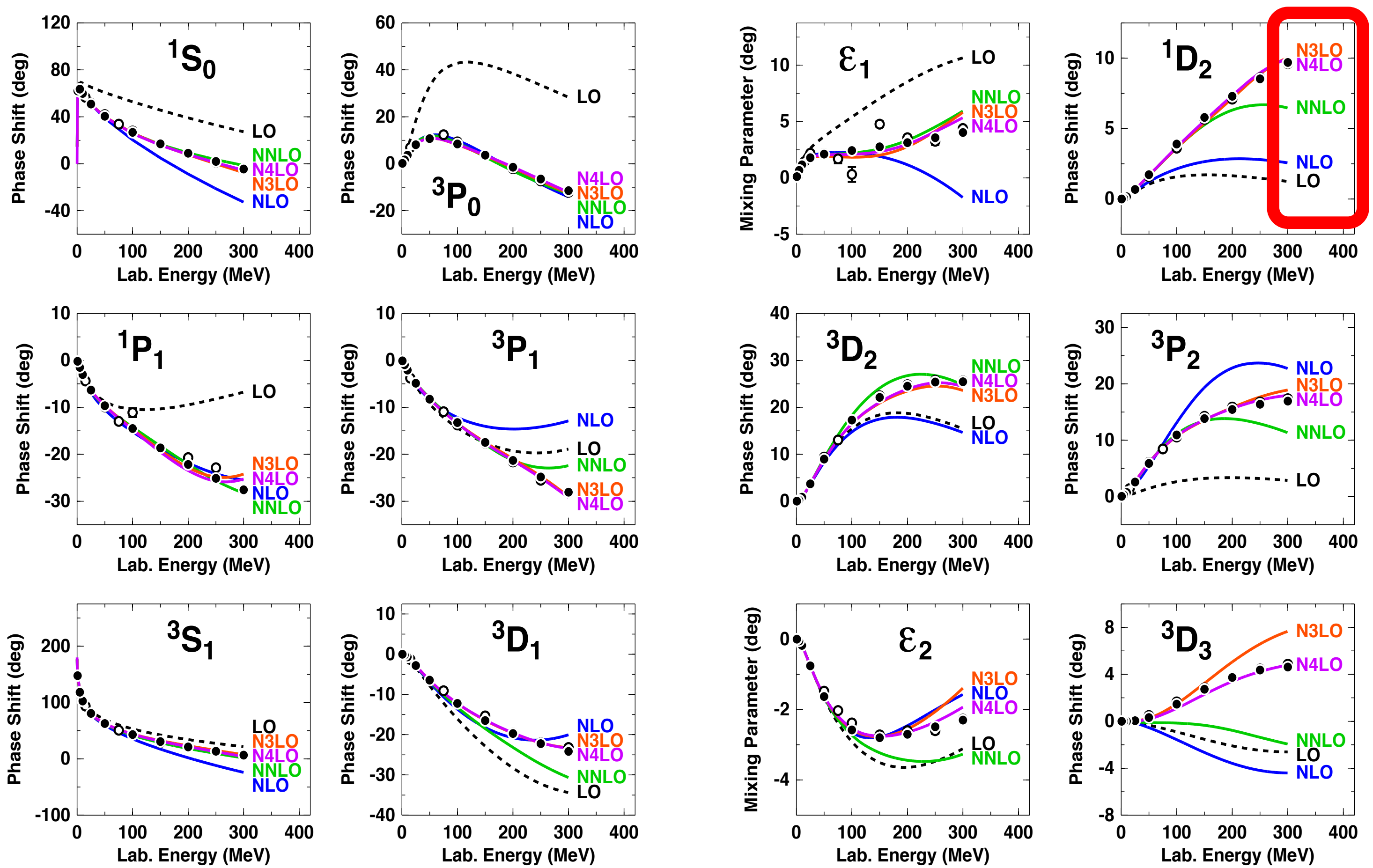
- **Use π-N LECs determined in π-N analysis with the highest possible precision: Roy-Steiner Analysis (Hoferichter et al., PRL 115, 192301 (2015)).**
- **NN potentials are fit to NN data (and not to phase shifts) using all NN data below pion production threshold published up to December 2016.**

Reproduction of the NN Data

TABLE V: χ^2/datum for the fit of the 2016 NN data base y NN potentials at various orders of chiral EFT ($\Lambda = 500$ MeV in all cases).

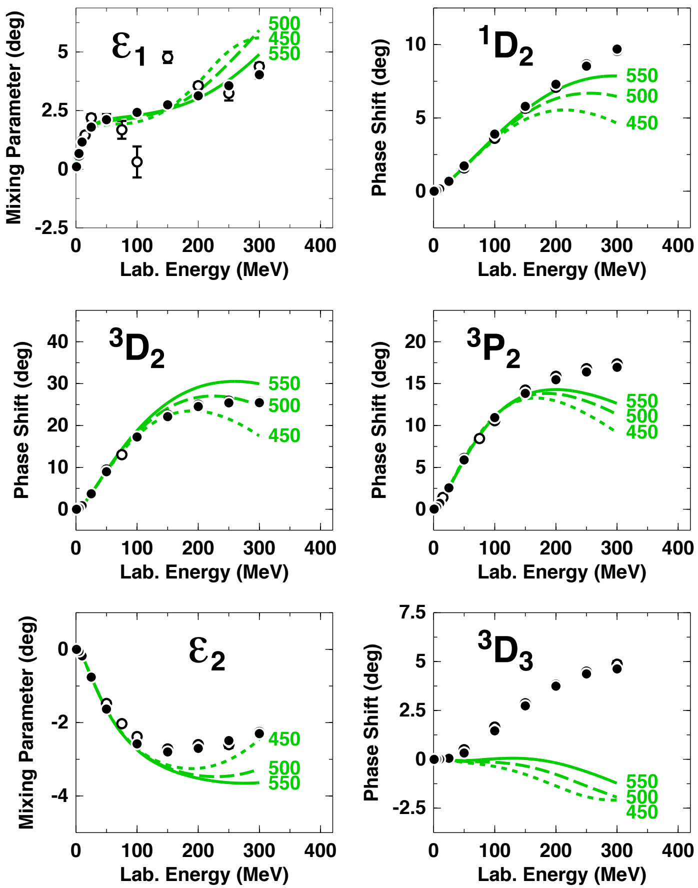
$T_{\rm lab}$ bin (MeV)	No. of data	LO	NLO	NNLO	N^{3}LO	N^{4}LO
			proton-proton			
0–100	795	520	18.9	2.28	1.18	(Includes ct's In F-waves.)
0–190	1206	430	43.6	4.64	1.69	1.12
0–290	2132	360	70.8	7.60	2.09	1.21
			neutron-proton			
0–100	1180	114	7.2	1.38	0.93	0.94
0–190	1697	96	23.1	2.29	1.10	1.06
0–290	2721	94	36.7	5.28	1.27	1.10
			pp* plus *np			
0–100	1975	283	11.9	1.74	1.03	1.00
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
0–290	4853	206	51.5	6.30	1.63	1.15

Neutron-Proton Phase Shifts

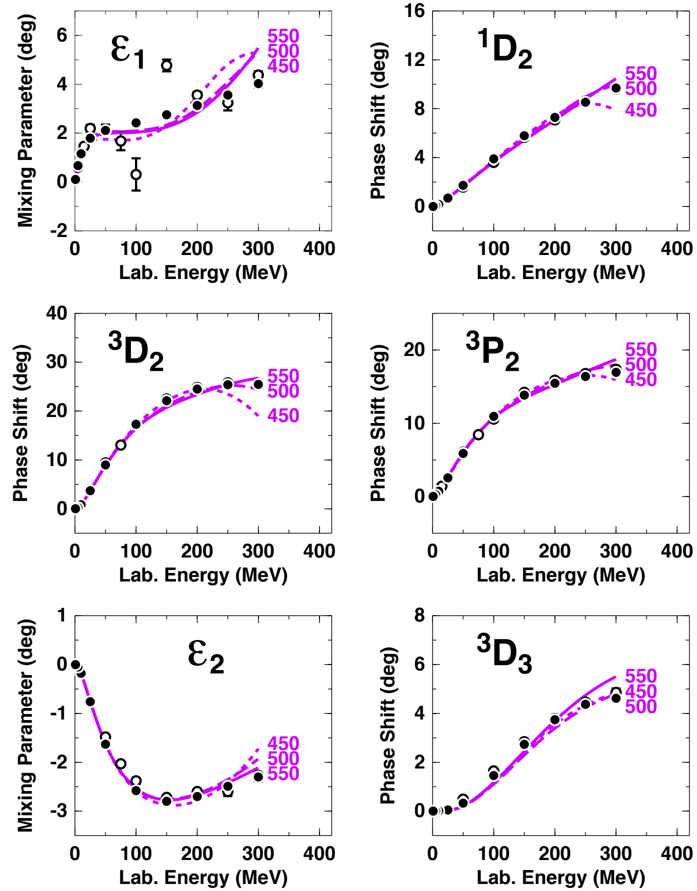


Cutoff Variations

NNLO



N4LO



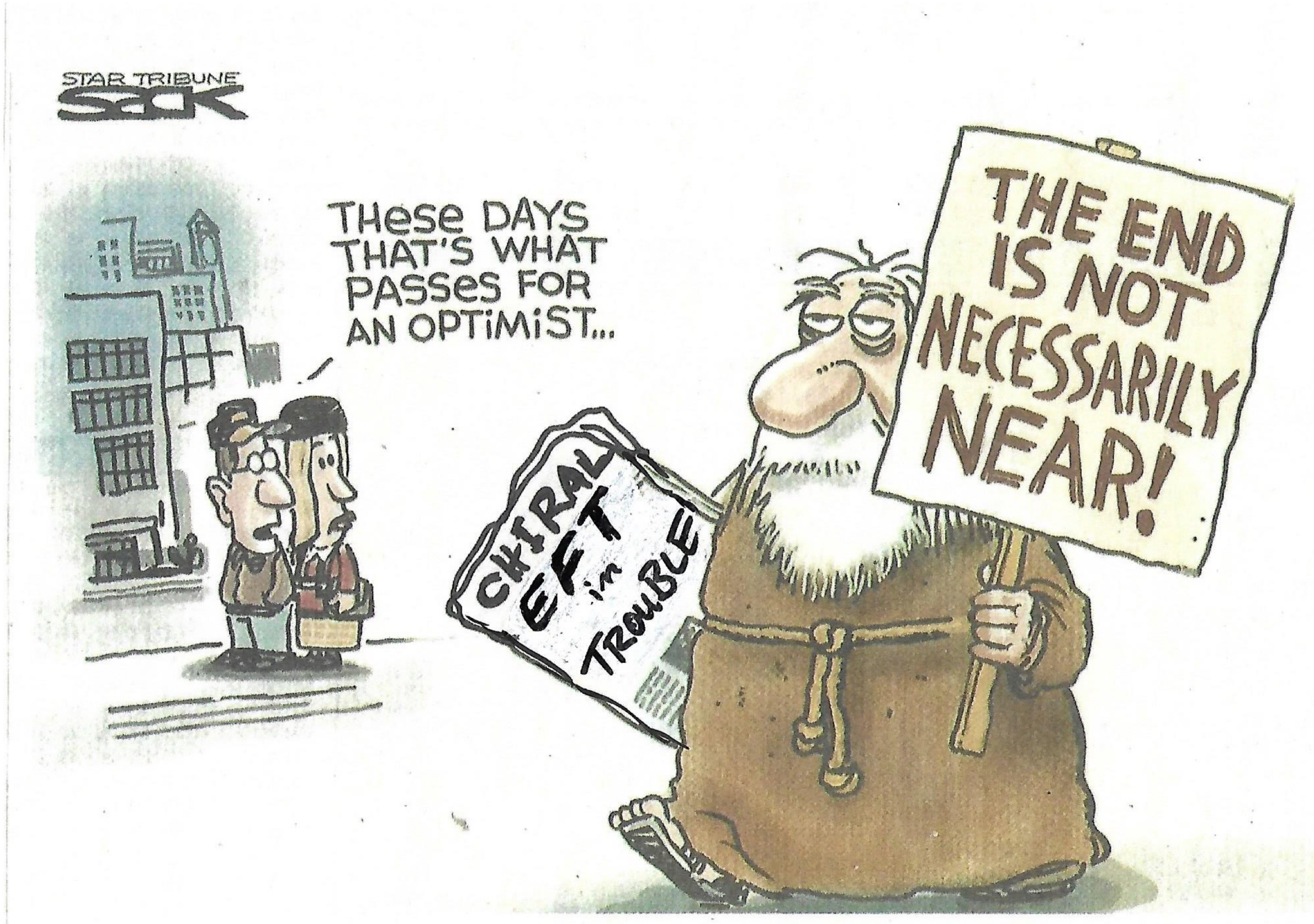
The Potentials are non-local and soft

TABLE VII: Two- and three-nucleon bound-state properties as predicted by NN potentials at various orders of chiral EFT ($\Lambda = 500$ MeV in all cases). (Deuteron: Binding energy B_d, asymptotic S state A_S, asymptotic D/S state η, structure radius $r_{\rm str}$, quadrupole moment Q, D-state probability P_D; the predicted $r_{\rm str}$ and Q are without meson-exchange current contributions and relativistic corrections. Triton: Binding energy B_t.) B_d is fitted, all other quantities are predictions.

	LO	NLO	NNLO	N^{3}LO	N^{4}LO	Empirical[a]
Deuteron						
B_d (MeV)	2.224575	2.224575	2.224575	2.224575	2.224575	2.224575(9)
A_S (fm$^{-1/2}$)	0.8526	0.8828	0.8844	0.8853	0.8852	0.8846(9)
η	0.0302	0.0262	0.0257	0.0257	0.0258	0.0256(4)
$r_{\rm str}$ (fm)	1.911	1.971	1.968	1.970	1.973	1.97507(78)
Q (fm^2)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
P_D (%)	7.29	3.40	4.49	4.15	4.10	—
Triton						
B_t (MeV)	11.02	8.31	8.21	8.09	8.08	8.48

CONCLUSIONS

- Concerning the *ab initio* explanation of intermediate and heavy nuclei we are faced with tough issues.
- But, let's not (yet) give up on the systematic use of chiral EFT.
- This requires order-by-order calculations up to N4LO using consistent 2NF and 3NF (and 4NF).
- **For this purpose, we have constructed a family of NN potentials that keeps the error budget as low as possible: Essentially no uncertainties in the π-N LECs (Roy-Steiner!), Accurate fit to the 2016 NN data base (≅5000 data).**
- The NN potentials are relatively soft and require less 3NF as compared to some other chiral NN potentials that are floating around (like, locals, "semi-locals").
- Systematic calculations with different families of chiral interactions may hopefully give us clues for how to solve the remaining problems.


STAR TRIBUNE
THESE DAYS THAT'S WHAT PASSES FOR AN OPTIMIST...
THE END IS NOT NECESSARILY NEAR!
CHIRAL EFT in TROUBLE!

But, one farther day,

The End

will come. Be patient.